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Solve the following inequality and graph the solution set

Contents: This page corresponds to § 2.5 (p. 216) of the text. Suggested Problems from Text: p. 225 #11, 12, 13, 14, 16, 28, 33, 35, 38, 41, 53, 56, 62, 63, 68, 69 Linear Inequalities An inequality is a comparison of expressions by either "less than" (=). Note that Html does not support the standard symbols for "less than or equal to" and "greater than or equal to", so we use = for these relations. Example 1. x + 3 (7)(-2). This is TRUE. So, when we multiply the original inequality by -2, we must reverse the direction to obtain another true statement. Note: In general we may not multiply or divide both sides of an inequality by an expression with a variable, because some values of the variable may make the expression positive and some may make it negative. Example 3. 7 - 2x < 3. -2x < -4. x > 2. Note: When we divided both sides of the inequality by -2 we changed the direction of the inequality. Look at the graphs of the functions on either side of the inequality. To satisfy the inequality, 7 - 2x needs to be less than 3. So we are looking for numbers x such that the point on the graph of y = 7 - 2x is below the point on the graph of y = 3. This is true for x > 2. In interval notation the solution set is (2, inf). There is another way to use a graphing utility to solve this inequality. In the Java Grapher the expression (7-2*x)L3 has the value 1 for numbers x that satisfy the inequality, and the value 0 for other numbers x. The picture below shows the graph of (7-2*x)L3 as drawn by the Grapher. Exercise 1: Solve the inequality 4 - x > 1 + 3x. Answer Return to Contents Combinations of Inequalities Example 4. Find all numbers x such that -3 < 5 - 2x and 5 - 2x < 9. -3 < 5 - 2x -8 < -2x 4 > x (-inf, 4) AND 5 - 2x < 9 -2x < 4 x > -2 (-2, inf) In order to satisfy both inequalities, a number must be in both solution sets. So the numbers that satisfy both inequalities are the values in the intersection of the two solution sets, which is the set (-2, 4) in interval notation. The problem above is usually written as a double inequality. -3 < 5 - 2x < 9 stands for -3 < 5 - 2x and 5 - 2x < 9. Note: When we solved the two inequalities separately, the steps in the two problems were the same. Therefore, the double inequality notation may be used to solve the inequalities simultaneously. -3 < 5 - 2x < 9. -8 < -2x < 4. 4 > x > -2. In terms of graphs, this problem corresponds to finding the values of x such that the corresponding point on the graph of y = 5 - 2x is between the graphs of y = -3 and y = 9. Example 5. Find all numbers x such that x + 1 < 0 or x + 1 > 3. In Example 4 above we were looking for numbers that satisfied both inequalities. Here we want to find the numbers that satisfy either of the inequalities. This corresponds to a union of solution sets instead of an intersection. Do not use the double inequality notation in this situation. x + 1 < 0 x < -1 (-inf, -1) OR x + 1 > 3 x > 2 (2, inf) The solution set is the union of the two intervals (-inf, -1) and (2, inf). Exercise 2: (a) 1 < 3 + 5x < 7 Answer (b) 2 - x < 1, or 2 - x > 5 Answer Return to Contents Inequalities Involving Absolute Values Inequalities involving absolute values can be rewritten as combinations of inequalities. Let a be a positive number. |x| < a if and only if -a < x < a. |x| > a if and only if x < -a or x > a. To make sense of these statements, think about a number line. The absolute value of a number is the distance the number is from 0 on the number line. So the inequality |x| < a is satisfied by numbers whose distance from 0 is less than a. This is the set of numbers between -a and a. The inequality |x| > a is satisfied by numbers whose distance from 0 is larger than a. This means numbers that are either larger than a, or less than -a. Example 6. | 3 + 2x | 0, so the graph of y = x2 - x - 6 is above the x-axis on the entire interval. Since we are looking for regions where the graph is below the axis, the solution set is -2 < x < 3, or (-2, 3). Common Mistake We will use the problem in Example 8 to illustrate a common mistake. x2 - x - 6 < 0. (x + 2)(x - 3) < 0 OK to this point. x + 2 < 0 or x - 3 < 0 WRONG! When a product of two numbers is equal to 0, then at least one of the numbers must be 0. However, a product of two negative numbers is not negative, so this approach is not useful for solving inequalities. Example 9. 1.2 x3 + 3.07 x2 - x - 3.71 > 0. This problem is much more difficult than the inequality in the previous example! It is not easy to factor, so we will not be able to find the exact values of the critical numbers. We will use a graphing utility to approximate the critical numbers. The graph of the polynomial is shown below. y = 1.2 x3 + 3.07 x2 - x - 3.71 The critical numbers are approximately -2.35, -1.25, and 1.05. In this problem we looking for regions where the graph is above the axis. Solution Set: (-2.35, -1.25) union (1.05, inf). Exercise 4: Solve the inequality x2 + 3x - 4 > 0. Use a graphing utility to check your solution. Return to Contents Rational Inequalities A rational expression is one of the form polynomial divided by polynomial. In general, graphs of rational functions do have breaks. They are not defined at the zeros of the denominator. These are the only places where there are breaks, so we can use the same technique to solve rational inequalities that we use for polynomial inequalities. Example 10. The critical numbers for a rational inequality are all the zeros of the numerator and the denominator. Since the numerator and denominator are already factored in this example, we see that the critical numbers are -3, 5, and 1. The three critical numbers divide the number line into four test intervals. (-inf, -3): -4 is in the interval, and the rational function evaluated at -4 is -9/15. Since the value is negative, the graph of the rational function is below the x-axis throughout the interval. (-3, 1): 0 is in the interval. The value of the function at 0 is 5, which is positive. The graph of the function is above the x-axis throughout the interval. (1, 5): 2 is in the interval. The value at 2 is -5. The graph of the function is below the x-axis. (5, inf): 6 is in the interval. The value at 6 is 9/15. The graph of the function is above the x-axis. We are looking for regions where the graph is above the x-axis, so the solution set is (-3, 1) union (5, inf). Note: A graphing utility can be used to see which side of the x-axis the graph is on over the various test intervals. In some cases you must solve algebraically to find the exact values of the critical numbers, but once this is done, a grapher provides a fast way to finish the problem. Graph of y = (x + 3)(x - 5)/3(x - 1) There are two important points to keep in mind when working with inequalities: 1. We need to compare an expression to 0. So, if we start with the problem x2 - 3x - 11 < x + 10, we would subtract x and 10 from both sides to obtain x2 - 4x - 21 < 0. 2. Do not multiply both sides of an inequality by an expression with a variable. For example, given the problem , do not multiply both sides by x. The correct way to handle this problem is as follows: Now we see that the critical numbers are 0 (from denominator), 1, and -1. Exercise 5: (a) Finish solving x2 - 3x - 11 < x + 10, and check your solution with a graphing utility. (b) Finish solving , and check your solution with a graphing utility. Return to Contents Learning Outcomes Describe solutions to inequalities Represent inequalities on a number line Represent inequalities using interval notation Solve single-step inequalities Use the addition and multiplication properties to solve algebraic inequalities and express their solutions graphically and with interval notation Solve inequalities that contain absolute value Solve multi-step inequalities Combine properties of inequality to isolate variables, solve algebraic inequalities, and express their solutions graphically Simplify and solve algebraic inequalities using the distributive property to clear parentheses and fractions First, let’s define some important terminology. An inequality is a mathematical statement that compares two expressions using the ideas of greater than or less than. Special symbols are used in these statements. When you read an inequality, read it from left to right—just like reading text on a page. In algebra, inequalities are used to describe large sets of solutions. Sometimes there are an infinite amount of numbers that will satisfy an inequality, so rather than try to list off an infinite amount of numbers, we have developed some ways to describe very large lists in succinct ways. The first way you are probably familiar with—the basic inequality. For example: [latex]{x}\lt{9}[/latex] indicates the list of numbers that are less than 9. Would you rather write [latex]{x}\lt{9}[/latex] or try to list all the possible numbers that are less than 9? (hopefully, your answer is no) [latex]-5\le{t}[/latex] indicates all the numbers that are greater than or equal to [latex]-5[/latex]. Note how placing the variable on the left or right of the inequality sign can change whether you are looking for greater than or less than. For example: [latex]x\lt5[/latex] means all the real numbers that are less than 5, whereas; [latex]5\lt{x}[/latex] means that 5 is less than x, or we could rewrite this with the x on the left: [latex]x\gt{5}[/latex] note how the inequality is still pointing the same direction relative to x. This statement represents all the real numbers that are greater than 5, which is easier to interpret than 5 is less than x. The second way is with a graph using the number line: And the third way is with an interval. We will explore the second and third ways in depth in this section. Again, those three ways to write solutions to inequalities are: an inequality an interval a graph Inequality Signs The box below shows the symbol, meaning, and an example for each inequality sign. Sometimes it’s easy to get tangled up in inequalities, just remember to read them from left to right. Symbol Words Example [latex]eq [/latex] not equal to [latex]{2}eq{8}[/latex], 2 is not equal to 8. [latex]\gt[/latex] greater than [latex]{5}\gt{1}[/latex], 5 is greater than 1 [latex]\lt[/latex] less than [latex]{2}\lt{11}[/latex], 2 is less than 11 [latex] \geq [/latex] greater than or equal to [latex]{4}\geq{ 4}[/latex], 4 is greater than or equal to 4 [latex]\leq [/latex] less than or equal to [latex]{7}\leq{9}[/latex], 7 is less than or equal to 9 The inequality [latex]x>y[/latex] can also be written as [latex]{y}) or less than (b[/latex], then [latex]a+c>b+c[/latex]. If [latex]a>b[/latex], then [latex]a−c>b−c[/latex]. Because inequalities have multiple possible solutions, representing the solutions graphically provides a helpful visual of the situation, as we saw in the last section. The example below shows the steps to solve and graph an inequality and express the solution using interval notation. The line represents all the numbers to which you can add 3 and get a number that is less than 5. There’s a lot of numbers that solve this inequality! Just as you can check the solution to an equation, you can check a solution to an inequality. First, you check the end point by substituting it in the related equation. Then you check to see if the inequality is correct by substituting any other solution to see if it is one of the solutions. Because there are multiple solutions, it is a good practice to check more than one of the possible solutions. This can also help you check that your graph is correct. The example below shows how you could check that [latex]x-17[/latex] The previous examples showed you how to solve a one-step inequality with the variable on the left hand side. The following video provides examples of how to solve the same type of inequality. What would you do if the variable were on the right side of the inequality? In the following example, you will see how to handle this scenario. Solve for x: [latex]4\geq{x}+5[/latex] Check the solution to [latex]4\geq{x}+5[/latex] The following video show examples of solving inequalities with the variable on the right side. Solve inequalities with multiplication and division Solving an inequality with a variable that has a coefficient other than 1 usually involves multiplication or division. The steps are like solving one-step equations involving multiplication or division EXCEPT for the inequality sign. Let’s look at what happens to the inequality when you multiply or divide each side by the same number. Let’s start with the true statement: [latex]10>5[/latex] Let’s try again by starting with the same true statement: [latex]10>5[/latex] Next, multiply both sides by the same positive number: [latex]10\cdot 2>5\cdot 2[/latex] This time, multiply both sides by the same negative number: [latex]10\cdot-2>5 \\ \,\,\,\,\,\,\cdot -2\,\cdot-2[/latex] 20 is greater than 10, so you still have a true inequality: [latex]20>10[/latex] Wait a minute! [latex]−20[/latex] is not greater than [latex]−10[/latex], so you have an untrue statement. [latex]−20>−10[/latex] When you multiply by a positive number, leave the inequality sign as it is! You must “reverse” the inequality sign to make the statement true: [latex]−20b[/latex] [latex]c[/latex] [latex]ac>bc[/latex] [latex]a>b[/latex] [latex]-c[/latex] [latex]acb[/latex] [latex]c[/latex] [latex] \displaystyle \frac{a}{c}>\frac{b}{c}[/latex] [latex]a>b[/latex] [latex]-c[/latex] [latex] \displaystyle \frac{a}{c}-12x[/latex] The following video gives examples of how to solve an inequality with the multiplication property of equality where the variable is on the right hand side. Combine properties of inequality to solve algebraic inequalities A popular strategy for solving equations, isolating the variable, also applies to solving inequalities. By adding, subtracting, multiplying and/or dividing, you can rewrite the inequality so that the variable is on one side and everything else is on the other. As with one-step inequalities, the solutions to multi-step inequalities can be graphed on a number line. Solve for x: [latex]3x–7\ge 41[/latex] Check the solution. When solving multi-step equations, pay attention to situations in which you multiply or divide by a negative number. In these cases, you must reverse the inequality sign. In the following video, you will see an example of solving a linear inequality with the variable on the left side of the inequality, and an example of switching the direction of the inequality after dividing by a negative number. In the following video, you will see an example of solving a linear inequality with the variable on the right side of the inequality, and an example of switching the direction of the inequality after dividing by a negative number. Simplify and solve algebraic inequalities using the distributive property As with equations, the distributive property can be applied to simplify expressions that are part of an inequality. Once the parentheses have been cleared, solving the inequality will be straightforward. In the following video, you are given an example of how to solve a multi-step inequality that requires using the distributive property. In the next example, you are given an inequality with a term that looks complicated. If you pause and think about how to use the order of operations to solve the inequality, it will hopefully seem like a straightforward problem. Use the textbox to write down what you think is the best first step to take. Solve for a. [latex] \displaystyle\frac{{2}{a}-{4}}{{6}}{