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Determining the intervals where a function is increasing or decreasing is a fundamental concept in calculus. Understanding this helps visualize the function's behavior and is crucial for various applications, including optimization problems. This guide provides a step-by-step approach to finding these intervals. Understanding Increasing and Decreasing Functions A function is said to be increasing on an interval if its values consistently rise as the input (x-values) increases within that interval. Conversely, a function is decreasing if its values consistently fall as the input increases. Visually, an increasing function slopes upward from left to right, while a decreasing function slopes downward. Steps to Find Increasing and Decreasing Intervals Find the First Derivative: The key to identifying increasing and decreasing intervals lies in the function's first derivative, f'(x). The first derivative represents the instantaneous rate of change of the function at any given point. Find Critical Points: Critical points occur where the first derivative is either zero (f'(x) = 0) or undefined. These points are potential turning points where the function might transition from increasing to decreasing or vice-versa. Analyze the Sign of the First Derivative: This is the crucial step. We need to examine the sign (positive or negative) of the first derivative in the intervals created by the critical points. f'(x) > 0: If the first derivative is positive in an interval, the function is increasing in that interval. f'(x) < 0: If the first derivative is negative in an interval, the function is decreasing in that interval. Write the Intervals: Finally, express the increasing and decreasing intervals using interval notation. For example, (a, b) represents the interval from 'a' to 'b', excluding 'a' and 'b'. [a, b] includes 'a' and 'b'. Use parentheses for intervals where the function is increasing or decreasing up to, but not including, a critical point. Example Let's find the increasing and decreasing intervals for the function $f(x) = x^3 - 3x^2 + 2$. First Derivative: $f'(x) = 3x^2 - 6x$ Critical Points: Set f'(x) = 0: $3x^2 - 6x = 0$ This factors to 3x(x - 2) = 0, giving critical points at x = 0 and x = 2. Analyze the Sign of the First Derivative: Interval (-∞, 0): Choose a test point, say x = -1. $f'(-1) = 3(-1)^2 - 6(-1) = 9 > 0$. Therefore, f(x) is increasing on (-∞, 0). Interval (0, 2): Choose a test point, say x = 1. $f'(1) = 3(1)^2 - 6(1) = -3 < 0$. Therefore, f(x) is decreasing on (0, 2). Interval (2, ∞): Choose a test point, say x = 3. $f'(3) = 3(3)^2 - 6(3) = 9 > 0$. Therefore, f(x) is increasing on (2, ∞). Intervals: Increasing: (-∞, 0) and (2, ∞) Decreasing: (0, 2) Conclusion By following these steps, you can effectively determine the increasing and decreasing intervals of a function. Remember that understanding the behavior of the first derivative is key to unlocking this important aspect of function analysis. This method provides a powerful tool for comprehending function behavior and solving related problems. Increasing and decreasing functions refer to the behavior of a function's graph as you move from left to right along the x-axis. A function is considered increasing if for any two values x1 and x2 such that x1 < x2 , the function value at x1 is less than the function value at x2 (i.e., f(x1) < f(x2)). On the other hand, a function is decreasing if f(x1) > f(x2) for x1 < x2.Some real-life examples of increasing/decreasing functions are given below:Increasing Function DefinitionIn simple words, an increasing function is a type of function where, with increasing input (or the independent variable), output also increases (or the value of the function). Now, let's define an increasing function formally.Now, let us consider I to be an interval that is present in the domain of a real-valued function f, then the function f is increasing on I.if x1 < x2 ⇒ f(x1) ≤ f(x2) ∀ x1 and x2 ∈ ISome common examples of increasing functions include linear functions with positive slope (such as y = mx + b), exponential functions (such as y = ax, where a is a positive constant), and power functions (such as y = xn, where n is a positive integer).Solved Example: Consider the function: f(x) = 2x + 3. If x1 = 2 and x2 = 5. Determine whether the function is increasing or decreasing.Solution:A function is considered increasing if, for any two values x1 and x2 such that we have f(x1) < f(x2).Given that x1 = 2, x2 = 5.f(x1) = f(2) = 2(2) + 3 = 4 + 3 = 7f(x2) = f(5) = 2(5) + 3 = 10 + 3 = 13Since 7 ≤ 13, the function satisfies the definition of an increasing function. Therefore, f(x) = 2x + 3 is increasing.Strictly Increasing FunctionFor a function to be strictly increasing, the function should be increasing, but it can't be equal for any two unequal values, i.e.,if x1 < x2 ⇒ f(x1) < f(x2) ∀ x1 and x2 ∈ IDecreasing Function DefinitionIn simple words, a decreasing function is a type of function where, with increasing input (or the independent variable), the output value decreases (or the value of the function). To define a decreasing function formally, let us consider I to be an interval that is present in the domain of a real-valued function f, then the function f is decreasing on I.if x1 < x2 ⇒ f(x1) ≥ f(x2) ∀ x1 and x2 ∈ ISome common examples of decreasing functions include exponential decay functions (such as y = a^(-x), where a is a positive constant), and negative power functions (such as y = x^(-n), where n is a positive integer).Solved Example: Determine whether the function f(x) = -2x + 5 is decreasing, if x1 = 1 and x2 = 3.Solution:A function is decreasing if f(x1) > f(x2) for x1 < x2.Since x1 < x2, we now calculate f(x1) and f(x2):f(x1) = f(1) = −2(1) + 5 = −2 + 5 = 3f(x2) = f(3) = -2(3) + 5 = -6 + 5 = -1Clearly, f(1) > f(3) So as x increases, f(x) decreases. This satisfies the condition for a strictly decreasing function.Strictly Decreasing FunctionFor a function to be strictly decreasing, the function should be decreasing, but it can't be equal for any two unequal values, i.e.,if x1 < x2 ⇒ f(x1) > f(x2) ∀ x1 and x2 ∈ IConstant Function DefinitionIn simple words, a constant function is a type of function where, regardless of the input or independent variable, the output always remains the same, i.e., for all the inputs output remains constant. To define a constant function more formally, a function f is said to be a constant function if and only iff(x) = kWhere k is the real number.➢ Read all about functions in Maths - [Read Here!]Rules to Check Increasing and Decreasing FunctionsIn calculus, an increasing function can be defined in terms of the slope of any curve, as an increasing function always has a positive slope, i.e., dy/dx > 0. To define an increasing function more formally, let us consider f to be a function that is continuous on the interval [p, q] and differentiable on the open interval (p, q), thenFunction f is increasing in [p, q] if f'(x) > 0 for each x ∈ (p, q).As a decreasing function always has a negative slope, a decreasing function can be defined in terms of the slope of any curve, i.e., dy/dx < 0. For a more formal definition of the decreasing function, let us consider f to be a function that is continuous on the interval [p, q] and differentiable on the open interval (p, q), thenFunction f is decreasing in [p, q] if f'(x) < 0 for each x ∈ (p, q).Graph of Increasing, Decreasing, and Constant FunctionsThe graphical representation of an increasing function, a decreasing function, and a constant function is,Example: In this example, we will investigate the graph of f(x) = x2. Solution:Function table: x-4-3-2-101234f(x)16941014916As we can see that when x < 0, the value of f(x) is decreasing as the graph moves to the right. In other words, the "height" of the graph is getting smaller. This is also confirmed by looking at the table of values. When x < 0, as x increases, f(x) decreases. Therefore, f(x) is decreasing on the interval from negative infinity to 0. When x > 0, the opposite is happening. When x > 0, the value of f(x) is increasing as the graph moves to the right. In other words, the "height" of the graph is getting bigger. This is also confirmed by looking at the table of values. When x > 0, as x increases, f(x) increases. Therefore, f(x) is increasing on the interval from 0 to infinity. Properties of Increasing & Decreasing FunctionsSome helpful algebraic properties of increasing & Decreasing Functions are as follows:Additive property. If the functions f and g are increasing/decreasing on the interval (a, b), then the sum of the functions f + g is also increasing/decreasing on this interval.Opposite property. If the function f is increasing/decreasing on the interval (a, b), then the opposite function, -f, is decreasing/increasing.Inverse property. If the function f is increasing/decreasing on the interval (a, b), then the inverse function, 1/f, is decreasing/increasing on this interval.Multiplicative property. If the functions f and g are increasing/decreasing and not negative on the interval (a, b), then the product of the functions is also increasing/decreasing.How to Find Increasing and Decreasing IntervalsGiven a function, f(x), we can determine the intervals where it is increasing and decreasing by using differentiation and algebra. Step 1: Find the derivative, f'(x), of the function. Step 2: Find the zeros of f'(x). Remember, zeros are the values of x for which f'(x) = 0. Set f'(x) = 0 and solve for x. Step 3: Determine the intervals. The intervals are between the endpoints of the interval of f(x) and the zeros of f'(x). If the interval of f(x) is not given, assume f(x) is on the interval (-∞, ∞). Step 4: Determine whether the function is increasing or decreasing on each interval. Given the interval (a, c), choose a value b, a < b < c. Solve for f'(b). If f'(b) is positive, f(x) is increasing on (a, c). If f'(b) is negative, f(x) is decreasing on (a, c).Example 1: If g(x) = (x - 5)2, find the intervals where g(x) is increasing and decreasing. Solution:Step 1: Find the derivative of the function. Using the chain rule, g'(x) = 2(5 - x)Step 2: Find the zeros of the derivative function. In other words, find the values of for which g(x) equals zero. You can do this by setting g(x) = 0 and using algebra to solve for x. From the definitions above, we know the function is constant at points where the derivative is zero. g'(x) = 0 = 2(5 - x) ⇒ 0 = 5 - x⇒ x = 5Step 3: Use the zeros to determine intervals.Since x = 5 is the only zero for g'(x), there are just 2 intervals: from negative infinity to 5, and from 5 to negative infinity. These can be denoted in inequality notation: -∞ < x < 5⇒ 5 < x < ∞Or in interval notation: (-∞, 5), (5, ∞)Remember, the endpoints are NOT inclusive because g(x) is neither increasing nor decreasing at the endpoints. Step 4: Determine if the function is increasing or decreasing in each interval.For the first interval, ((-∞, 5), we'll choose b = 0. -∞ < x < 5g'(b) = g'(0) = 2(5-0) = 10 10 > 0 POSITIVEFor the second interval, (5, ∞), we'll choose b = 6. 5 < 6 < ∞g'(b) = g'(6) = 2(5-6) = -2 -2 < 0 NEGATIVETherefore, g(x) is increasing on (-∞, 5) and decreasing on (5, ∞). We can verify our results visually. In the graph below, you can clearly see that f(x) = (x - 5)2 is increasing on the interval (5, ∞) and decreasing on the interval (-∞, 5).We can visually verify our result by investigating the graph of g(x). Looking at the graph, g(x) is indeed increasing in the interval from negative infinity to 5 and decreasing in the interval from 5 to infinity. Example 2: Find the intervals in -20 < x < 20 where g(x) is increasing and decreasing, given g'(x) = x2 - 100.Solution:If the derivative is given, we can skip the first step and go straight to finding the zeroes. g'(x)= 0 = x2 - 100⇒ x2 = 100⇒ x = 10, -10Intervals: (-20, -10), (-10, 10), (10, 20) For (-20, -10), we'll choose b = -12. -20 < -12 < -10 g'(-12) = 44 > 0 For (-10, 10), we'll choose b = 0. -10 < 0 < 10 g(0) = -100 < 0 For (10, 20), we'll choose b = 12. 10 < 12 < 20 g(12) = 44 > 0Hence, for -20 < x < 20, g(x) is increasing on (-20, -10) and (10, 20) and decreasing on (-10, 10). Read More,Types of FunctionsGraphing of FunctionSolved Question on Increasing and Decreasing FunctionsQuestion 1: Given the function g(x) = 3x2 - 12, find the intervals on -3 < x < 3 where g(x) is increasing and decreasing. Solution:Given function: g(x) = 3x2 - 12Differentiate w.r.t. x, we get g'(x) = 6x For increasing and decreasing Put g'(x) = 0 ⇒ g'(x) = 6x = 0⇒ x = 0 Intervals: (-3, 0), (0, 3) At x = -2, g'(-2) = -12 < 0 At x = 2, g'(2) = 12 > 0 So, for -3 < x < 3, g(x) is decreasing on (-3, 0) and increasing on (0, 3). Question 2: Given the derivative of f(x), f'(x) = -10x2 + 40x, find the intervals where f(x) is increasing and decreasing. Solution:Given: f'(x) = -10x2 + 40xFor increasing and decreasing Put f'(x) = 0 ⇒ f'(x) = -10x2 + 40x = 0⇒ x = 4, 0Intervals: (−∞, 0), (0, 4), (4, ∞) So, at x = -1, f'(-1) = -50 < 0 at x = 1, f'(1) = 30 > 0 at x = 5, f'(5) = -50 < 0 So, f(x) is increasing on (0, 4) and decreasing on (−∞, 0), (4, ∞)Question 3: Given the function g(x) = 5x2 - 20x + 100, find the intervals where g(x) is increasing and decreasing. Solution:Given: g(x) = 5x2 - 20x + 100⇒ g'(x) = 10x - 20 For increasing and decreasing Put g'(x) = 0 ⇒ g'(x) = 10x - 20 = 0⇒ x = 2 Intervals: (−∞, 2), (2, ∞)At, x = 1, g'(1) = -10 < 0 At x = 3, g'(3) = 10 > 0 So, g(x) is decreasing on (-∞, 2) and increasing on (2, ∞) Question 4: Given the functions s(x) = 6x3 - x2, find the intervals on 0 < x < 10 where s(x) is increasing and decreasing. Solution:Given: s(x) = 6x3 - x2⇒ s'(x) = 18x2 - 2x For increasing and decreasing Put s'(x) = 0 ⇒ s'(x) = 18x2 - 2x = 0⇒ x = 1/9, -1/9Intervals: (0, 1/9) Here, -1/9 is not in the given interval, 0 < x < 10 For 0 < x < 1/9: Choose x=0.05:s'(0.05) = 18(0.05)2 - 2(0.05) = 0.045 - 0.1 = -0.055So, s(x)s is decreasing on (0,1/9).For x > 1/9: Choose x=1:s'(1) = 18(1)2 - 2(1) = 18 - 2 = 16So, s(x) is increasing on (1/9,10).Question 5:Solution:Given: g'(x) = 7x2 - 8For increasing and decreasing Put g'(x) = 0 ⇒ g'(x) = 7x2 - 8 = 0 ⇒ x = {√(8/7), -√(8/7)}Intervals: (-∞,√(8/7)), (-√(8/7), √(8/7)), (√(8/7), ∞)So, At x = -10, g'(-10) = 692 > 0 At x = 0, g'(0) = -8 < 0 At x = 10, g'(10) = 692 < 0 Hence, g(x) is increasing on (-∞,√(8/7))and (√(8/7), ∞), and decreasing on (-√(8/7), √(8/7))Practice MorePractice Questions on FunctionsQuiz on Functions Increasing and decreasing intervals are intervals of real numbers where the real-valued functions are increasing and decreasing respectively. To determine the increasing and decreasing intervals, we use the first-order derivative test to check the sign of the derivative in each interval. The interval is increasing if the value of the function f(x) increases with an increase in the value of x and it is decreasing if f(x) decreases with a decrease in x. In this article, we will learn to determine the increasing and decreasing intervals using the first-order derivative test and the graph of the function with the help of examples for a better understanding of the concept. What are Increasing and Decreasing Intervals? The intervals where the functions are increasing or decreasing are called the increasing and decreasing intervals. These intervals can be evaluated by checking the sign of the first derivative of the function in each interval. If the first derivative of a function is positive in an interval, then it is said to be an increasing interval and if the first derivative of the function is negative in an interval, then it is said to be a decreasing interval. Let us go through their formal definitions to understand their meaning: Increasing and Decreasing Intervals Definition The definitions for increasing and decreasing intervals are given below. For a real-valued function f(x), the interval I is said to be an increasing interval if for every x < y, we have f(x) ≤ f(y). For a real-valued function f(x), the interval I is said to be a decreasing interval if for every x < y, we have f(x) ≥ f(y). We can also define the increasing and decreasing intervals using the first derivative of the function f(x) as: If f'(x) ≥ 0 on I, then I is said to be an increasing interval. If f'(x) ≤ 0 on I, then I is said to be a decreasing interval. Finding Increasing and Decreasing Intervals Now, we have understood the meaning of increasing and decreasing intervals, let us now learn how to do calculate increasing and decreasing intervals of functions. We will solve an example to understand the concept better. Consider f(x) = x3 + 3x2 - 45x + 9. Differentiate f(x) with respect to x to find f'(x). f'(x) = 3x2 + 6x - 45 = 3(x2 + 2x - 15) = 3 (x + 5) (x - 3) Substitute f'(x) = 0 ⇒ x = -5, x = 3 Now, the x-intercepts are of f'(x) are x = -5 and x = 3. The intervals that we have are (-∞, -5), (-5, 3), and (3, ∞). We will check the sign of f'(x) in each of these intervals to identify increasing and decreasing intervals. Interval Value of x f'(x) Increasing/Decreasing (-∞, -5) x = -6 f'(-6) = 27 > 0 Increasing (-5, 3) x = 0 f'(0) = -45 < 0 Decreasing (3, ∞) x = 4 f'(4) = 27 > 0 Increasing Hence, the increasing intervals for f(x) = x3 + 3x2 - 45x + 9 are (-∞, -5) and (3, ∞), and the decreasing interval of f(x) is (-5, 3). Increasing and Decreasing Intervals Using Graph We have learned to identify the increasing and decreasing intervals using the first derivative of the function. Now, we will determine the intervals just by seeing the graph. Given below are samples of two graphs of different functions. The first graph shows an increasing function as the graph goes upwards as we move from left to right along the x-axis. The second graph shows a decreasing function as the graph moves downwards as we move from left to right along the x-axis. Important Notes on Increasing and Decreasing Intervals For a real-valued function f(x), the interval I is said to be a strictly increasing interval if for every x < y, we have f(x) < f(y). For a real-valued function f(x), the interval I is said to be a strictly decreasing interval if for every x < y, we have f(x) > f(y). The function is constant in an interval if f'(x) = 0 through that interval. Related Topics Applications of Derivatives Differential Equations Calculus Example 1: Determine the increasing and decreasing intervals for the function f(x) = -x3 + 3x2 + 9. Solution: Differentiate f(x) = -x3 + 3x2 + 9 w.r.t. x f'(x) = -3x2 + 6x = -3x(x - 2) ⇒ f'(x) = 0 ⇒ -3x(x - 2) = 0 ⇒ x = 0, or x = 2 The intervals that we have are (-∞, 0), (0, 2), and (2, ∞). We need to identify the increasing and decreasing intervals from these. Interval Value of x f'(x) Increasing/Decreasing (-∞, 0) x = -1 f'(-1) = -9 < 0 Decreasing (0, 2) x = 1 f'(1) = 3 > 0 Increasing (2, ∞) x = 4 f'(4) = -24 < 0 Decreasing Answer: Hence, (-∞, 0) and (2, ∞) are decreasing intervals, and (0, 2) are increasing intervals. Example 2: Show that (-∞, ∞) is a strictly increasing interval for f(x) = 3x + 5. Solution: Consider two real numbers x and y in (-∞, ∞) such that x < y. Then, we have x < y ⇒ 3x < 3y ⇒ 3x + 5 < 3y + 5 ⇒ f(x) < f(y) Since x and y are arbitrary, therefore f(x) < f(y) whenever x < y. Answer: Hence, (-∞, ∞) is a strictly increasing interval for f(x) = 3x + 5. View Answer > go to slidego to slide Breakdown tough concepts through simple visuals. Math will no longer be a tough subject, especially when you understand the concepts through visualizations. Book a Free Trial Class FAQs on Increasing and Decreasing Intervals Increasing and decreasing intervals are intervals of real numbers where the real-valued functions are increasing and decreasing respectively. Why are Only the X-values Used in Describing Increasing and Decreasing Intervals? X-values are used to describe increasing and decreasing intervals because the values of the function f(x) increases or decreases with the increase in the x-values, i.e., the change in f(x) is dependent on the value of x. How Do You Find Increasing and Decreasing Intervals of a Function? We can find increasing and decreasing intervals of a function using its first derivative. We can find the critical points and hence, the intervals. Then, we can check the sign of the derivative in each interval to identify increasing and decreasing intervals. How to Find Increasing and Decreasing Intervals Using Graph? We can find increasing and decreasing intervals using a graph by seeing if the graph moves upwards or downwards as moves from left to right along the x-axis. For graphs moving upwards, the interval is increasing and if the graph is moving downwards, the interval is decreasing. How Do You Know When a Function is Increasing? A function f(x) is said to be increasing on an interval I if for any two numbers x and y in I such that x < y, we have f(x) ≤ f(y). Which Function Does Not Have Increasing and Decreasing Intervals? A constant function is neither increasing nor decreasing as the graph of a constant function is a straight line parallel to the x-axis and its derivative is always 0. Determine if the following is true or false. $$f(x)=x^2 - 3x$$ Decreasing on: $$\left(-\infty, \frac{3}{2}\right)$$ Increasing on: $$\left(\frac{3}{2}, \infty\right)$$ Please choose the best answer. As part of exploring how functions change, we can identify intervals over which the function is changing in specific ways. We say that a function is increasing on an interval if the function values increase as the input values increase within that interval. Similarly, a function is decreasing on an interval if the function values decrease as the input values increase over that interval. The average rate of change of an increasing function is positive, and the average rate of change of a decreasing function is negative. Figure 3 shows examples of increasing and decreasing intervals on a function. Figure 3. The function [latex]f\left(x\right)={x}^{3}-12x[/latex] is increasing on [latex]\left(-\infty \text{,}-\text{2}\right){\cup }^{\text{ }}^{\text{ }}\left(2,\infty \right)[/latex] and is decreasing on [latex]\left(-2\text{,}2\right)[/latex]. This video further explains how to find where a function is increasing or decreasing. While some functions are increasing (or decreasing) over their entire domain, many others are not. A value of the input where a function changes from increasing to decreasing (as we go from left to right, that is, as the input variable increases) is called a local maximum. If a function has more than one, we say it has local maxima. Similarly, a value of the input where a function changes from decreasing to increasing as the input variable increases is called a local minimum. The plural form is “local minima.” Together, local maxima and minima are called local extrema, or local extreme values, of the function. (The singular form is “extremum.”) Often, the term local is replaced by the term relative. In this text, we will use the term local. Clearly, a function is neither increasing nor decreasing on an interval where it is constant. A function is also neither increasing nor decreasing at extrema. Note that we have to speak of local extrema, because any given local extremum as defined here is not necessarily the highest maximum or lowest minimum in the function’s entire domain. For the function in Figure 4, the local maximum is 16, and it occurs at [latex]x=-2[/latex]. The local minimum is [latex]-16[/latex] and it occurs at [latex]x=2[/latex]. Figure 4 To locate the local maxima and minima from a graph, we need to observe the graph to determine where the graph attains its highest and lowest points, respectively, within an open interval. Like the summit of a roller coaster, the graph of a function is higher at a local maximum than at nearby points on both sides. The graph will also be lower at a local minimum than at neighboring points. Figure 5 illustrates these ideas for a local maximum. Figure 5. Definition of a local maximum. These observations lead us to a formal definition of local extrema. A function [latex]f[/latex] is an increasing function on an open interval if [latex]f\left(b\right)>f\left(a\right)[/latex] for any two input values [latex]a[/latex] and [latex]b[/latex] in the given interval where [latex]b>a[/latex]. A function [latex]f[/latex] is a decreasing function on an open interval if [latex]f\left(b\right)a[/latex]. A function [latex]f[/latex] has a local maximum at [latex]x=b[/latex] if there exists an interval [latex]\left(a,c\right)[/latex] with [latex]a Given the function [latex]p\left(t\right)[/latex] in the graph below, identify the intervals on which the function appears to be increasing. Figure 6 We see that the function is not constant on any interval. The function is increasing where it slants upward as we move to the right and decreasing where it slants downward as we move to the right. The function appears to be increasing from [latex]t=1[/latex] to [latex]t=3[/latex] and from [latex]t=4[/latex] on. In interval notation, we would say the function appears to be increasing on the interval (1,3) and the interval [latex]\left(4,\infty \right)[/latex]. Graph the function [latex]f\left(x\right)=\frac{2}{x}+\frac{x}{3}[/latex]. Then use the graph to estimate the local extrema of the function and to determine the intervals on which the function is increasing. Using technology, we find that the graph of the function looks like that in Figure 7. It appears there is a low point, or local minimum, between [latex]x=2[/latex] and [latex]x=3[/latex], and a mirror-image high point, or local maximum, somewhere between [latex]x=-3[/latex] and [latex]x=-2[/latex]. Figure 7 Graph the function [latex]f\left(x\right)={x}^{3}-6{x}^{2}-15x+20\\[/latex] to estimate the local extrema of the function. Use these to determine the intervals on which the function is increasing and decreasing. Solution For the function [latex]f[/latex] whose graph is shown in Figure 9, find all local maxima and minima. Figure 9 Observe the graph of [latex]f[/latex]. The graph attains a local maximum at [latex]x=1[/latex] because it is the highest point in an open interval around [latex]x=1[/latex]. The local maximum is the [latex]y[/latex] -coordinate at [latex]x=1[/latex], which is [latex]2[/latex]. The graph attains a local minimum at [latex]\text{ }x=-1\text{ }[/latex] because it is the lowest point in an open interval around [latex]x=-1[/latex]. The local minimum is the y-coordinate at [latex]x=-1[/latex], which is [latex]-2[/latex]. We will now return to our toolkit functions and discuss their graphical behavior in the table below. Function Increasing/Decreasing Example Constant Function [latex]f\left(x\right)={c}[/latex] Neither increasing nor decreasing   Identity Function [latex]f\left(x\right)={x}[/latex] Increasing Quadratic Function [latex]f\left(x\right)={x}^{2}[/latex] Increasing on [latex]\left(0,\infty\right)[/latex] Decreasing on [latex]\left(-\infty,0\right)[/latex] Minimum at [latex]x=0[/latex] Cubic Function [latex]f\left(x\right)={x}^{3}[/latex] Increasing Reciprocal [latex]f\left(x\right)=\frac{1}{x}[/latex] Decreasing [latex]\left(-\infty,0\right)\cup\left(0,\infty\right)[/latex] Reciprocal Squared [latex]f\left(x\right)=\frac{1}{{x}^{2}}[/latex] Increasing on [latex]\left(-\infty,0\right)[/latex] Decreasing on [latex]\left(0,\infty\right)[/latex] Cube Root [latex]f\left(x\right)=\sqrt[3]{x}[/latex] Increasing   Square Root [latex]f\left(x\right)=\sqrt{x}[/latex] Increasing on [latex]\left(0,\infty\right)[/latex]   Absolute Value [latex]f\left(x\right)=|x|[/latex] Increasing on [latex]\left(0,\infty\right)[/latex] Decreasing on [latex]\left(-\infty,0\right)[/latex]   As part of exploring how functions change, we can identify intervals over which the function is changing in specific ways. We say that a function is increasing on an interval if the function values increase as the input values increase within that interval. Similarly, a function is decreasing on an interval if the function values decrease as the input values increase over that interval. The average rate of change of an increasing function is positive, and the average rate of change of a decreasing function is negative. Figure 3 shows examples of increasing and decreasing intervals on a function. Figure 3 The function \(f(x)=x^{3}-12 x\) is increasing on \((-\infty,-2) \cup(2, \infty)\) and is decreasing on \((-2,2)\). While some functions are increasing (or decreasing) over their entire domain, many others are not. A value of the input where a function changes from increasing to decreasing (as we go from left to right, that is, as the input variable increases) is the location of a local maximum. The function value at that point is the local maximum. If a function has more than one, we say it has local maxima. Similarly, a value of the input where a function changes from decreasing to increasing as the input variable increases is the location of a local minimum. The function value at that point is the local minimum. The plural form is "local minima". Together, local maxima and minima are called local extrema, or local extreme values, of the function. (The singular form is "extremum"). Often, the term local is replaced by the term relative. In this text, we will use the term local. Clearly, a function is neither increasing nor decreasing on an interval where it is constant. A function is also neither increasing nor decreasing at extrema. Note that we have to speak of local extrema, because any given local extremum as defined here is not necessarily the highest maximum or lowest minimum in the function's entire domain. For the function whose graph is shown in Figure 4, the local maximum is 16, and it occurs at \(x=-2\). The local minimum is \(-16\) and it occurs at \(x=2\). Figure 4 To locate the local maxima and minima from a graph, we need to observe the graph to determine where the graph attains its highest and lowest points, respectively, within an open interval. Like the summit of a roller coaster, the graph of a function is higher at a local maximum than at nearby points on both sides. The graph will also be lower at a local minimum than at neighboring points. Figure 5 illustrates these ideas for a local maximum. Figure 5 Definition of a local maximum LOCAL MINIMA AND LOCAL MAXIMA A function \(f\) is an increasing function on an open interval if \(f(b) > f(a)\) for any two input values \(a\) and \(b\) in the given interval where \(b > a\). A function \(f\) is a decreasing function on an open interval if \(f(b) < f(a)\) for any two input values \(a\) and \(b\) in the given interval where \(b > a\). A function \(f\) has a local maximum at \(x=b\) if there exists an interval \((a, c)\) with \(a < b < c\) such that, for any \(x\) in the interval \((a, c), f(x) \leq f(b)\). Likewise, \(f\) has a local minimum at \(x=b\) if there exists an interval \((a, c)\) with \(a < b < c\) such that, for any \(x\) in the interval \((a, c), f(x) \geq f(b)\). EXAMPLE 7 Finding Increasing and Decreasing Intervals on a Graph Given the function \(p(t)\) in Figure 6, identify the intervals on which the function appears to be increasing. Figure 6 SolutionWe see that the function is not constant on any interval. The function is increasing where it slants upward as we move to the right and decreasing where it slants downward as we move to the right. The function appears to be increasing from \(t=1\) to \(t=3\) and from \(t=4\) on.In interval notation, we would say the function appears to be increasing on the interval \((1,3)\) and the interval \((4, \infty)\). Analysis Notice in this example that we used open intervals (intervals that do not include the endpoints), because the function is neither increasing nor decreasing at \(t=1, t=3\), and \(t=4\). These points are the local extrema (two minima and a maximum). EXAMPLE 8 Finding Local Extrema from a Graph Graph the function \(f(x)=\dfrac{2}{x}+\dfrac{x}{3}\). Then use the graph to estimate the local extrema of the function and to determine the intervals on which the function is increasing. SolutionUsing technology, we find that the graph of the function looks like that in Figure 7 . It appears there is a low point, or local minimum, between \(x=2\) and \(x=3\), and a mirror-image high point, or local maximum, somewhere between \(x=-3\) and \(x=-2\).Figure 7 Analysis Most graphing calculators and graphing utilities can estimate the location of maxima and minima. Figure 8 provides screen images from two different technologies, showing the estimate for the local maximum and minimum. Figure 8 Based on these estimates, the function is increasing on the interval \((-\infty,-2.449)\) and \((2.449, \infty)\). Notice that, while we expect the extrema to be symmetric, the two different technologies agree only up to four decimals due to the differing approximation algorithms used by each. (The exact location of the extrema is at \(\pm \sqrt{6}\), but determining this requires calculus). TRY IT #4 Graph the function \(f(x)=x^{3}-6 x^{2}-15 x+20\) to estimate the local extrema of the function. Use these to determine the intervals on which the function is increasing and decreasing. EXAMPLE 9 Finding Local Maxima and Minima from a Graph For the function \(f\) whose graph is shown in Figure 9, find all local maxima and minima. Figure 9 Solution Observe the graph of \(f\). The graph attains a local maximum at \(x=1\) because it is the highest point in an open interval around \(x=1\). The local maximum is the \(y\)-coordinate at \(x=1\), which is 2. The graph attains a local minimum at \(x=-1\) because it is the lowest point in an open interval around \(x=-1\). The local minimum is the \(y\)-coordinate at \(x=-1\), which is \(-2\).