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[illegible]



Subla Sūtras has been described as "the earliest extant verbal expression of the Pythagorean Theorem in the world, although it had already been known to the Old Babylonians." [8] They make use of Pythagorean triples.[9][10] which are particular cases of Diophantine equations.[11] According to mathematician S. G. Dani, the Babylonian cuneiform tablet Plimpton 322 written c. 1850 BC[12] "contains fifteen Pythagorean triples with whole largentries, called triads, including (13500, 12709, 18541) which is a primitive triple,[13] indicating, in particular, that there was sophisticated understanding on the topic" in Mesopotamia until 1850 BC.[14] "Since these tablets predate the Sulbasūtras period by several centuries, taking into account the contextual appearance of some of the triples, it is reasonable to expect that similar understanding would have been there in India." [14] Dani goes on to say:[15] As the main objective of the Sulvasūtras was to describe the constructions of altars and the geometric principles involved in them, the subject of Pythagorean triples, even if it had been well understood may still not have featured in the Sulvasūtras. The occurrence of the triples in the Sulvasūtras is comparable to mathematics that one may encounter in an introductory book on architecture or another similar applied area, and would not correspond directly to the overall knowledge on the topic at that time. Since, unfortunately, no other contemporaneous sources have been found it may never be possible to settle this issue satisfactorily. See also: Greek mathematics Pythagorean theorem: a2 + b2 = c2 Thales (635-543 BC) of Miletus (now in southwesten Turkey), was the first to whom deduction in mathematics is attributed. There are five geometric propositions for which he wrote deductive proofs, though his proofs have not survived. Pythagoras (582–496 BC) of Ionia, and later, Italy, then colonized by Greeks, may have been a student of Thales, and traveled to Babylon and Egypt. The theorem that bears his name may not have been his discovery, but he was probably one of the first to give a deductive proof of it. He gathered a group of students around him to study mathematics, music, and philosophy, and together they discovered most of what high school students learn today in their geometry courses. In addition, they made the profound discovery of incommensurable lengths and irrational numbers. Plato (427–347 BC) was a philosopher, highly esteemed by the Greeks. There is a story that he had inscribed above the entrance to his famous school, "Let none ignorant of geometry enter here." However, the story is considered to be untrue.[16] Though he was not a mathematician himself, his views on mathematics had great influence. Mathematicians thus accepted his belief that geometry should use no tools but compass and straightedge - never measuring instruments such as a marked ruler or a protractor, because these were a workman's tools, not worthy of a scholar. This dictum led to a deep study of possible compass and straightedge constructions, and three classic construction problems: how to use these tools to trisect an angle, to construct a cube twice the volume of a given cube, and to construct a square equal in area to a given circle. The proofs of the impossibility of these constructions, finally achieved in the 19th century, led to important principles regarding the deep structure of the real number system. Aristotle (384-322 BC). Plato's greatest pupil, wrote a treatise on methods of reasoning used in deductive proofs (see Logic) which was not substantially improved upon until the 19th century. Statue of Euclid in the Oxford University Museum of Natural History Woman teaching geometry. Illustration at the beginning of a medieval translation of Euclid's Elements (c. 1310) Euclid (c. 325–265 BC), of Alexandria, probably a student at the Academy founded by Plato, wrote a treatise in 13 books (chapters), titled The Elements of Geometry, in which he presented geometry in an ideal axiomatic form, which came to be known as Euclidean geometry. The treatise is not a compendium of all that the Hellenistic mathematicians knew at the time about geometry; Euclid himself wrote eight more advanced books on geometry. We know from other references that Euclid's was not the first elementary geometry textbook, but it was so much superior that the others fell into disuse and were lost. He was brought to the university at Alexandria by Ptolemy I, King of Egypt. The Elements began with definitions of terms, fundamental geometric principles (called axioms or postulates), and general quantitative principles (called common notions) from which all the rest of geometry could be logically deduced. Following are his five axioms, somewhat paraphrased to make the English easier to read. Any two points can be joined by a straight line. Any finite straight line can be extended in a straight line. A circle can be drawn with any center and any radius. All right angles are equal to each other. If two straight lines in a plane are crossed by another straight line (called the transversal), and the interior angles between the two lines and the transversal lying on one side of the transversal add up to less than two right angles, then on that side of the transversal, the two lines extended will intersect (also called the parallel postulate). Concepts, that are now understood as algebra, were expressed geometrically by Euclid, a method referred to as Greek geometric algebra. Archimedes (287–212 BC), of Syracuse, Sicily, when it was a Greek city-state, was one of the most famous mathematicians of the Hellenistic period. He is known for his formulation of a hydrostatic principle (known as Archimedes' principle) and for his works on geometry, including Measurement of the Circle and On Conoids and Spheroids. His work on Floating Bodies is the first known work on hydrostatics, of which Archimedes is recognized as the founder. Renaissance translations of his works, including the ancient commentaries, were enormously influential in the work of some of the best mathematicians of the 17th century, notably René Descartes and Pierre de Fermat.[17] Geometry was connected to the divine for most medieval scholars. The compass in this 13th-century manuscript is a symbol of God's act of Creation. After Archimedes, Hellenistic mathematics began to decline. There were a few minor stars yet to come, but the golden age of geometry was over. Proclus (410-485), author of Commentary on the First Book of Euclid, was one of the last important players in Hellenistic geometry. He was a competent geometer, but more importantly, he was a superb commentator on the works that preceded him. Much of that work did not survive to modern times, and is known to us only through his commentary. The Roman Republic and Empire that succeeded and absorbed the Greek city-states produced excellent engineers, but no mathematicians of note. The great Library of Alexandria was later burned. There is a growing consensus among historians that the Library of Alexandria likely suffered from several destructive events, but that the destruction of Alexandria's pagan temples in the late 4th century was probably the most severe and final one. The evidence for that destruction is the most definitive and secure. Caesar's invasion may well have led to the loss of some 40,000-70,000 scrolls in a warehouse adjacent to the port (as Luciano Canfora argues, they were likely copies produced by the Library intended for export), but it is unlikely to have affected the Library or Museum, given that there is ample evidence that both existed later. [18] Civil wars, decreasing investments in maintenance and acquisition of new scrolls and generally declining interest in non-religious pursuits likely contributed to a reduction in the body of material available in the Library, especially in the 4th century. The Serapeum was certainly destroyed by Theophilus in 391, and the Museum and Library may have fallen victim to the same campaign. See also: Indian mathematics In the Bakshshali manuscript, there is a handful of geometric problems (including problems about volumes of irregular solids). The Bakshshali manuscript also "employs a decimal place value system with a dot for zero." [19] Aryabhata's Aryabhatiya (499) includes the computation of areas and volumes. Brahmagupta wrote his astronomical work Brāhma Sphuta Siddhānta in 628. Chapter 12, containing 66 Sanskrit verses, was divided into two sections: "basic operations" (including cube roots, fractions, ratio and proportion, and barter) and "practical mathematics" (including mixture, mathematical sciences, plane figures, stacking bricks, sawing of timber, and piling of grain).[20] In the latter section, he stated his famous theorem on the diagonals of a cyclic quadrilateral.[20] Brahmagupta's theorem: If a cyclic quadrilateral has diagonals that are perpendicular to each other, then the perpendicular line drawn from the point of intersection of the diagonals to any side of the quadrilateral always bisects the opposite side. Chapter 12 also included a formula for the area of a cyclic quadrilateral (a generalization of Heron's formula), as well as a complete description of rational triangles (i.e. triangles with rational sides and rational areas). Brahmagupta's formula: The area, A, of a cyclic quadrilateral with sides of lengths a, b, c, d, respectively, is given by 



A
=
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s
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a
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(
s
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b
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s
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c
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(
s
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d
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{\displaystyle A={\sqrt {(s-a)(s-b)(s-c)(s-d)}}}

 where s, the semiperimeter, given by: 



s
=



a
+
b
+
c
+
d

2





.


{\displaystyle s={\frac {a+b+c+d}{2}}.}

 Brahmagupta's Theorem on rational triangles: A triangle with rational sides a, b, c, 



{\displaystyle a,b,c}

 and rational area is of the form: 



a
=
u

2


v
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v
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b
=
u

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w
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w
,
 
c
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u

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v
+

u

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v
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u

2


w
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(
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+
w
)


{\displaystyle a={\frac {u^{2}}{v}}+v,\;b={\frac {u^{2}}{w}}+w,\;c={\frac {u^{2}}{v}}+{\frac {u^{2}}{w}}-{\sqrt {(v+w)}}}

 for some rational numbers u, v, 



{\displaystyle u,v,}

 and 



{\displaystyle v,w.}

 [21] Parameshvara Nambudri was the first mathematician to give a formula for the radius of the circle circumscribing a cyclic quadrilateral.[22] The expression is sometimes attributed to Lhuillier [1782], 350 years later. With the sides of the cyclic quadrilateral being a, b, c, and d, the radius R of the circumscribed circle is: 



R
=



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{\displaystyle R={\sqrt {(a+b+c+d)(ac+bd)(ad+bc)}\;{\frac {(a+b+c+d)(a+b+c+d)(a+b-c+d)(a+b-c-d)(a+b-c-d)(a+b-c-d)}{(a+b+c+d)(a+b+c+d)(a+b+c+d)(a+b+c+d)(a+b+c+d)(a+b+c+d)}}}

. See also: Chinese mathematics The Nine Chapters on the Mathematical Art, first compiled in 179 AD, with added commentary in the 3rd century by Liu Hui Haidao Suanjing, Liu Hui, 3rd century The first definitive work (or at least oldest existent) on geometry in China was the Mo Jing, the Mohist canon of the early philosopher Mozi (470-390 BC). It was compiled years after his death by his followers around the year 330 BC.[23] Although the Mo Jing is the oldest existent book on geometry in China, there is the possibility that even older written material existed. However, due to the infamous Burning of the Books in a political maneuver by the Qin dynasty ruler Qin Shihuang (r. 221-210 BC), multitudes of written literature created before his time were purged. In addition, the Mo Jing presents geometrical concepts in mathematics that are perhaps too advanced not to have had a previous geometrical base or mathematic background to work upon. The Mo Jing described various aspects of many fields associated with physical science, and provided a small wealth of information on mathematics as well. It provided an 'atomic' definition of the geometric point, stating that a line is separated into parts, and the part which has no remaining parts (i.e. cannot be divided into smaller parts) and thus forms the extreme end of a line is a point.[23] Much like Euclid's first and third definitions and Plato's 'beginning of a line', the Mo Jing stated that "a point may stand at the end (of a line) or at its beginning like a head-presentation in childbirth. (As to its invisibility) there is nothing similar to it." [24] Similar to the atomists of Democritus, the Mo Jing stated that a point is the smallest unit, and cannot be cut in half, since 'nothing' cannot be halved.[24] It stated that two lines of equal length will always finish at the same place.[24] While providing definitions for the comparion of lengths and for parallels,[25] along with principles of space and bounded space.[26] [16] Civil wars, decreasing investments in maintenance and acquisition of new scrolls and generally declining interest in non-religious pursuits likely contributed to a reduction in the body of material available in the Library, especially in the 4th century. The Serapeum was certainly destroyed by Theophilus in 391, and the Museum and Library may have fallen victim to the same campaign. See also: Indian mathematics In the Bakshshali manuscript, there is a handful of geometric problems (including problems about volumes of irregular solids). The Bakshshali manuscript also "employs a decimal place value system with a dot for zero." [19] Aryabhata's Aryabhatiya (499) includes the computation of areas and volumes. Brahmagupta wrote his astronomical work Brāhma Sphuta Siddhānta in 628. Chapter 12, containing 66 Sanskrit verses, was divided into two sections: "basic operations" (including cube roots, fractions, ratio and proportion, and barter) and "practical mathematics" (including mixture, mathematical sciences, plane figures, stacking bricks, sawing of timber, and piling of grain).[20] In the latter section, he stated his famous theorem on the diagonals of a cyclic quadrilateral.[20] Brahmagupta's theorem: If a cyclic quadrilateral has diagonals that are perpendicular to each other, then the perpendicular line drawn from the point of intersection of the diagonals to any side of the quadrilateral always bisects the opposite side. Chapter 12 also included a formula for the area of a cyclic quadrilateral (a generalization of Heron's formula), as well as a complete description of rational triangles (i.e. triangles with rational sides and rational areas). Brahmagupta's formula: The area, A, of a cyclic quadrilateral with sides of lengths a, b, c, d, respectively, is given by 



A
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c
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s
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d
)


{\displaystyle A={\sqrt {(s-a)(s-b)(s-c)(s-d)}}}

 where s, the semiperimeter, given by: 



s
=



a
+
b
+
c
+
d

2





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{\displaystyle s={\frac {a+b+c+d}{2}}.}

 Brahmagupta's Theorem on rational triangles: A triangle with rational sides a, b, c, 



{\displaystyle a,b,c}

 and rational area is of the form: 



a
=
u

2


v
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v
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b
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u

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w
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w
,
 
c
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u

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v
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u

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v
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u

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w
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(
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)


{\displaystyle a={\frac {u^{2}}{v}}+v,\;b={\frac {u^{2}}{w}}+w,\;c={\frac {u^{2}}{v}}+{\frac {u^{2}}{w}}-{\sqrt {(v+w)}}}

 for some rational numbers u, v, 



{\displaystyle u,v,}

 and 



{\displaystyle v,w.}

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R
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{\displaystyle R={\sqrt {(a+b+c+d)(ac+bd)(ad+bc)}\;{\frac {(a+b+c+d)(a+b+c+d)(a+b-c+d)(a+b-c-d)(a+b-c-d)(a+b-c-d)}{(a+b+c+d)(a+b+c+d)(a+b+c+d)(a+b+c+d)(a+b+c+d)(a+b+c+d)}}}

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c
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d
)


{\displaystyle A={\sqrt {(s-a)(s-b)(s-c)(s-d)}}}

 where s, the semiperimeter, given by: 



s
=



a
+
b
+
c
+
d

2





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{\displaystyle a,b,c}

 and rational area is of the form: 



a
=
u

2


v
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v
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b
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u

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w
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w
,
 
c
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u

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{\displaystyle u,v,}

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R
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A
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s
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d
)


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s
=



a
+
b
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2





.


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 Brahmagupta's Theorem on rational triangles: A triangle with rational sides a, b, c, 



{\displaystyle a,b,c}

 and rational area is of the form: 



a
=
u

2


v
+
v
,
 
b
=
u

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w
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w
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c
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R
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{\displaystyle R={\sqrt {(a+b+c+d)(ac+bd)(ad+bc)}\;{\frac {(a+b+c+d)(a+b+c+d)(a+b-c+d)(a+b-c-d)(a+b-c-d)(a+b-c-d)}{(a+b+c+d)(a+b+c+d)(a+b+c+d)(a+b+c+d)(a+b+c+d)(a+b+c+d)}}}

. See also: Chinese mathematics The Nine Chapters on the Mathematical Art, first compiled in 179 AD, with added commentary in the 3rd century by Liu Hui Haidao Suanjing, Liu Hui, 3rd century The first definitive work (or at least oldest existent) on geometry in China was the Mo Jing, the Mohist canon of the early philosopher Mozi (470-390 BC). It was compiled years after his death by his followers around the year 330 BC.[23] Although the Mo Jing is the oldest existent book on geometry in China, there is the possibility that even older written material existed. However, due to the infamous Burning of the Books in a political maneuver by the Qin dynasty ruler Qin Shihuang (r. 221-210 BC), multitudes of written literature created before his time were purged. In addition, the Mo Jing presents geometrical concepts in mathematics that are perhaps too advanced not to have had a previous geometrical base or mathematic background to work upon. The Mo Jing described various aspects of many fields associated with physical science, and provided a small wealth of information on mathematics as well. It provided an 'atomic' definition of the geometric point, stating that a line is separated into parts, and the part which has no remaining parts (i.e. cannot be divided into smaller parts) and thus forms the extreme end of a line is a point.[23] Much like Euclid's first and third definitions and Plato's 'beginning of a line', the Mo Jing stated that "a point may stand at the end (of a line) or at its beginning like a head-presentation in childbirth. (As to its invisibility) there is nothing similar to it." [24] Similar to the atomists of Democritus, the Mo Jing stated that a point is the smallest unit, and cannot be cut in half, since 'nothing' cannot be halved.[24] It stated that two lines of equal length will always finish at the same place.[24] While providing definitions for the comparison of lengths and for parallels,[25] along with principles of space and bounded space.[26] [16] Civil wars, decreasing investments in maintenance and acquisition of new scrolls and generally declining interest in non-religious pursuits likely contributed to a reduction in the body of material available in the Library, especially in the 4th century. The Serapeum was certainly destroyed by Theophilus in 391, and the Museum and Library may have fallen victim to the same campaign. See also: Indian mathematics In the Bakshshali manuscript, there is a handful of geometric problems (including problems about volumes of irregular solids). The Bakshshali manuscript also "employs a decimal place value system with a dot for zero." [19] Aryabhata's Aryabhatiya (499) includes the computation of areas and volumes. Brahmagupta wrote his astronomical work Brāhma Sphuta Siddhānta in 628. Chapter 12, containing 66 Sanskrit verses, was divided into two sections: "basic operations" (including cube roots, fractions, ratio and proportion, and barter) and "practical mathematics" (including mixture, mathematical sciences, plane figures, stacking bricks, sawing of timber, and piling of grain).[20] In the latter section, he stated his famous theorem on the diagonals of a cyclic quadrilateral.[20] Brahmagupta's theorem: If a cyclic quadrilateral has diagonals that are perpendicular to each other, then the perpendicular line drawn from the point of intersection of the diagonals to any side of the quadrilateral always bisects the opposite side. Chapter 12 also included a formula for the area of a cyclic quadrilateral (a generalization of Heron's formula), as well as a complete description of rational triangles (i.e. triangles with rational sides and rational areas). Brahmagupta's formula: The area, A, of a cyclic quadrilateral with sides of lengths a, b, c, d, respectively, is given by 



A
=
(
s
−
a
)
(
s
−
b
)
(
s
−
c
)
(
s
−
d
)


{\displaystyle A={\sqrt {(s-a)(s-b)(s-c)(s-d)}}}

 where s, the semiperimeter, given by: 



s
=



a
+
b
+
c
+
d

2





.


{\displaystyle s={\frac {a+b+c+d}{2}}.}

 Brahmagupta's Theorem on rational triangles: A triangle with rational sides a, b, c, 



{\displaystyle a,b,c}

 and rational area is of the form: 



a
=
u

2


v
+
v
,
 
b
=
u

2


w
+
w
,
 
c
=
u

2


v
+

u

2


v
+
u

2


w
−
(
v
+
w
)


{\displaystyle a={\frac {u^{2}}{v}}+v,\;b={\frac {u^{2}}{w}}+w,\;c={\frac {u^{2}}{v}}+{\frac {u^{2}}{w}}-{\sqrt {(v+w)}}}

 for some rational numbers u, v, 



{\displaystyle u,v,}

 and 



{\displaystyle v,w.}

 [21] Parameshvara Nambudri was the first mathematician to give a formula for the radius of the circle circumscribing a cyclic quadrilateral.[22] The expression is sometimes attributed to Lhuillier [1782], 350 years later. With the sides of the cyclic quadrilateral being a, b, c, and d, the radius R of the circumscribed circle is: 



R
=



(
a
+
b
+
c
+
d
)
(
a
+
c
+
b
+
d
)
(
a
+
d
+
b
+
c
)
(
−
a
−
b
+
c
+
d
)
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a
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b
+
c
+
d
)
(
a
+
b
−
c
+
d
)


{\displaystyle R={\sqrt {(a+b+c+d)(ac+bd)(ad+bc)}\;{\frac {(a+b+c+d)(a+b+c+d)(a+b-c+d)(a+b-c-d)(a+b-c-d)(a+b-c-d)}{(a+b+c+d)(a+b+c+d)(a+b+c+d)(a+b+c+d)(a+b+c+d)(a+b+c+d)}}}

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A
=
(
s
−
a
)
(
s
−
b
)
(
s
−
c
)
(
s
−
d
)


{\displaystyle A={\sqrt {(s-a)(s-b)(s-c)(s-d)}}}

 where s, the semiperimeter, given by: 



s
=



a
+
b
+
c
+
d

2





.


{\displaystyle s={\frac {a+b+c+d}{2}}.}

 Brahmagupta's Theorem on rational triangles: A triangle with rational sides a, b, c, 



{\displaystyle a,b,c}

 and rational area is of the form: 



a
=
u

2


v
+
v
,
 
b
=
u

2


w
+
w
,
 
c
=
u

2


v
+

u

2


v
+
u

2


w
−
(
v
+
w
)


{\displaystyle a={\frac {u^{2}}{v}}+v,\;b={\frac {u^{2}}{w}}+w,\;c={\frac {u^{2}}{v}}+{\frac {u^{2}}{w}}-{\sqrt {(v+w)}}}

 for some rational numbers u, v, 



{\displaystyle u,v,}

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R
=



(
a
+
b
+
c
+
d
)
(
a
+
c
+
b
+
d
)
(
a
+
d
+
b
+
c
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−
a
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a
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b
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c
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(
a
+
b
−
c
+
d
)


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A
=
(
s
−
a
)
(
s
−
b
)
(
s
−
c
)
(
s
−
d
)


{\displaystyle A={\sqrt {(s-a)(s-b)(s-c)(s-d)}}}

 where s, the semiperimeter, given by: 



s
=



a
+
b
+
c
+
d

2





.


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 Brahmagupta's Theorem on rational triangles: A triangle with rational sides a, b, c, 



{\displaystyle a,b,c}

 and rational area is of the form: 



a
=
u

2


v
+
v
,
 
b
=
u

2


w
+
w
,
 
c
=
u

2


v
+

u

2


v
+
u

2


w
−
(
v
+
w
)


{\displaystyle a={\frac {u^{2}}{v}}+v,\;b={\frac {u^{2}}{w}}+w,\;c={\frac {u^{2}}{v}}+{\frac {u^{2}}{w}}-{\sqrt {(v+w)}}}

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R
=



(
a
+
b
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c
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a
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a
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