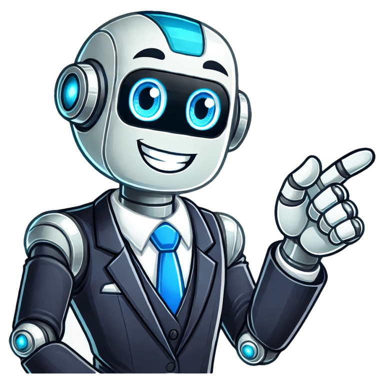

Simple Harmonic Motion is a fundament concept in the study of motion, especially oscillatory motion; which helps us understand many physical phenomena around like how strings produce pleasing sounds in a musical instrument such as the sitar, guitar, violin, etc., and also, how vibrations in the membrane in drums and diaphragms in telephone and speaker system creates the precise sound. Understanding Simple Harmonic Motion is key to understanding these phenomena. In this article, we will grasp the concept of Simple Harmonic Motion (SHM), its examples in real life, the equation, and how it is different from periodic motion. Simple Harmonic Motion Definition (SHM Definition)Simple harmonic motion is an oscillatory motion in which the acceleration of particle at any position is directly proportional to its displacement from the mean position. As SHM is an example of Oscillatory Motion. Simple Harmonic Motions (SHM) are all oscillatory and periodic, but not all Oscillatory or Periodic motions are SHM. Oscillatory motion is also referred to as Harmonic Motion and out of all Harmonic Motions, the most important one to study is Simple Harmonic Motion (SHM). Some characteristics of SHM are as follows: SHM is a type of Periodic and Oscillatory Motion.There is always a restoring force acting on an object in SHM, which always acts in the opposite direction to the displacement of the object from the mean position.The amplitude in the SHM remains constant throughout the motion of the object.The acceleration of the object is directly proportional to the displacement of the object from its mean position.The velocity of the object is maximum at the equilibrium position.The total energy in SHM remains conserved, as there is always a conversion of kinetic and potential energy happening throughout the motion.Examples of Simple Harmonic Motion(SHM)There are a lot of examples of Simple Harmonic Motion around us, we just need to see them from the perspective of SHM. From swings in the park to the motion of the cantilever, all are examples of SHM. The following illustration shows different examples of Simple Harmonic Motion. There are various terminologies related to SHM (Simple Harmonic Motion) some of which are explained as follows: Mean PositionIn Simple Harmonic Motion, the position of the object where there is no restoring force acting on it is the mean position. This is also the position about which the object moves between its extreme position is called the mean position of the object. The mean position is sometimes referred to as Equilibrium Position. AmplitudeThe amplitude in SHM is its maximum displacement from its equilibrium or mean position, and as displacement is a vector quantity, its direction is always away from the mean or equilibrium position. The SI unit of amplitude is the meter and all the other units of length can also be used for this. FrequencyThe frequency of SHM is the number of oscillations performed by a particle per unit of time. SI unit of frequency is Hertz or r.p.s. (rotations per second), and is given by: f = 1/T ω = 2f = 2π/T Time PeriodFor a particle performing SHM, the time period is the amount of time it takes to complete one complete oscillation. As a result, the time period or simply period of SHM is the shortest time before the motion repeats itself. T = 2π/ω where ω is the Angular frequency and T is the Time period. PhaseThe phase of SHM represents the magnitude and direction of particle displacement at any instant of the motion which is its state of oscillation. The expression for a particle's position as a function of time and angular frequency is as follows: x = A sin (ωt + ϕ) where (ωt + ϕ) is the phase of the particle. Phase DifferenceFor two particles performing SHM, the phase difference is defined as the difference between the total phase angles of those particles. Phase Difference is denoted by Δϕ. Mathematically the phase difference is defined as the difference between the total phase angles of two particles moving in simple harmonic motion with respect to the mean position. Suppose two particles performing SHM with the same angular frequency with displacement functions, x1 = A sin (ωt + ϕ1) and x2 = A sin (ωt + ϕ2). The phase difference is given by Δϕ = ϕ1 - ϕ2 When two vibrating particles with the same angular frequency, are in the same phase if the phase difference between them is an even multiple of π i.e., Δϕ = nπ Where, n = 0, 1, 2, 3, 4, . . . Two vibrating particles with the same angular frequency, are said to be in opposite phases if the phase difference between them is an odd multiple of π i.e., Δϕ = (2n + 1)π Where, n = 0, 1, 2, 3, 4, . . . Types of Simple Harmonic Motion (SHM)There are two types of SHM, which is: Linear Simple Harmonic MotionAngular Simple Harmonic MotionLinear Simple Harmonic MotionWhen a particle moves back and forth along a straight line with constant (called the equilibrium position), this is referred to as Linear SHM. Some examples of Linear SHM include the oscillation of a liquid column U-tube, the motion of a simple pendulum with very small displacements, and the vertical small vibration of a mass carried by elastic string. Conditions for Linear Simple Harmonic MotionThe restoring force or acceleration acting on the particle must always be proportional to the particle's displacement and directed toward the equilibrium position. F ∝ -x α ∝ -x where F is the Restoring ForceX is the Displacement of Particle from Equilibrium Positiona is the AccelerationAngular Simple Harmonic MotionAn angular simple harmonic motion occurs when a system oscillates angularly with respect to a fixed axis. The displacement of the particle in angular simple harmonic motion is measured in terms of angular displacement. The torsional pendulum is one example of Angular SHM. Conditions for Angular Simple Harmonic MotionThe restoring torque (or) angular acceleration acting on the particle should always be proportional to the particle's angular displacement and oriented towards the equilibrium position. T ∝ -θ α ∝ -θ where T is TorqueΘ is the Angular Displacementa is the Angular AccelerationDifference between Linear SHM and Angular SHMThere are some key differences between Linear and Angular SHM, some of which is as follows: In Linear SHM, the particle moves back and forth along a straight line with constant amplitude and frequency.In angular SHM, the particle moves in a circular path around an axis with a constant amplitude and frequency.Restoring Force is directly proportional to the linear displacement.Restoring torque is directly proportional to the angular displacement.Some examples of Linear SHM include a spring-mass system, a piston in the automobile, etc.Some examples of angular SHM include the motion of a pendulum, a rotating fan, or a balance wheel in a watch.Equations for Simple Harmonic MotionLet's consider a particle of mass (m) doing Simple Harmonic Motion along a path A'OA the mean position is O. Let the speed of the particle be V0 when it is at position P (at some distance from point O) At time, t = 0 the particle at P (moving towards point A) At time, t = t the particle is at Q (at a distance X from point O) at this point if velocity is V then: The force F acting on a particle at point p is given as, F = -K X [where, K = positive constant] We know that, F = m a [where, a = Acceleration at Q] ⇒ m a = -K x ⇒ a = -(K/m) x As K/m = ω2 Thus, a = -ω2x Also, we know that, a = d2X/d2t Therefore, d2X/d2t = -ω2x d2x/d2t + ω2x = 0 which is the differential equation for linear simple harmonic motion. Solutions of Differential Equations of SHMThe solutions to the differential equation for simple harmonic motion are as follows: Equation of SHM is, d2x/d2t + ω2x = 0 Multiplying by 2 \frac{dx}{dt}, to get 2 \frac{dx}{dt}\cdot \frac{d^2x}{dt^2} + 2 \omega^2 x \frac{dx}{dt} = 0 \Rightarrow \frac{d}{dt}\left(\left(\frac{dx}{dt}\right)^2+\omega^2 x^2\right)=0 After integration, we get a separable equation \left(\frac{dx}{dt}\right)^2+\omega^2 x^2=C^2 , \Rightarrow \frac{dx}{\sqrt{A^2-x^2}} \cdot dt =\omega \Rightarrow \frac{dx}{\sqrt{A^2-x^2}}=\omega dt Integrating, \sin^{-1}\left(\frac{x}{A}\right)=\omega t+\phi \Rightarrow \frac{x}{A} = \sin(\omega t+\phi) \bold{\Rightarrow x= A\sin(\omega t+\phi)} This is the required Solution of the SHM Equation. Different Cases of the Solution of SHM EquationFor particle is in the mean position at point (O) [ϕ =0], displacement function becomes x = Asinωt. For t = 0, when object is at point t, displacement function becomes x = Asinϕ For particle in any position throughout the SHM (any time t), displacement function becomes x = Asin(ωt+ϕ) Energy in Simple Harmonic Motion (SHM)A system performing SHM is called a Harmonic Oscillator. The energy in the particle performing simple harmonic motion is discussed below in the particle. Let's take a particle of mass (m) performing linear simple harmonic with angular frequency (ω) and the amplitude of the particle is (A) Now we know that the displacement of the particle at any time is given using the SHM equation, x = A sin (ωt + Φ)....(i) where Φ is the phase difference. Differentiating eq(i) wrt time we get, v = Aω.cos (ωt + Φ) v = Aω.Acos (ωt + Φ) ⇒ v = ω.√(A2 - x2)...(ii) Again, differentiating eq(ii) wrt time we get, a = -ω2. Asin (ωt + Φ) ⇒ a = -ω2x Restoring force acting on the body is, F = -kx where, k = mω2 Now for the energy of the particle performing SHM. Kinetic Energy of Particles in SHMKinetic Energy(K.E) = 1/2 mv2 {v2 = ω2(A2 - x2)} K.E = 1/2 mω2 (A2 - x2) Also, the kinetic energy of the particle in SHM is, K.E = 1/2 mω2 A2cos2(ωt + Φ) Potential Energy of Particles in SHMFor the potential energy we know that, Potential Energy(P.E) = - Work Done P.E = -F.dx P.E = kxdx (As dx is also negative) Integrating from o to x P.E = (kx2)/2 We know that, k = mω2 P.E = (mω2x2)/2 We know that, x = Asin(ωt + Φ)) P.E = (mω2)/2. A2sin2(ωt + Φ) Total Energy of the Particle in SHMTotal Energy(E) = Kinetic Energy(K.E) + Potential Energy(P.E) = 1/2 mω2(A2 - x2) + (mω2x2)/2 E = 1/2 mω2A2 This is the total energy of the particle in SHM. Simple Harmonic, Periodic, and Oscillation MotionIt seems that Simple Harmonic, Periodic, and Oscillation Motions are the same but they are indeed different. Now let's learn about them in detail. Simple Harmonic MotionThe motion of an object around a mean position in which the acceleration of the particle is directly proportional to the displacement of the particle is called Simple Harmonic Motion. Such as the motion of a cantilever. We can say that all Simple Harmonic Motions are oscillatory and periodic, but the converse is not true. Periodic MotionPeriodic motion is defined as the motion of any object that repeats its motion after a fixed interval of time. Such as the motion of the Moon around the Earth. Oscillation MotionOscillatory motion is the to-and-fro motion of an object from its mean position. SHM is an example of Oscillatory motion. Difference between Periodic, Oscillation, and Simple Harmonic MotionThere are some differences between Periodic motion, Oscillatory Motion, and Simple Harmonic Motion, which are listed as follows: The motion of an object is said to be periodic if it moves in such a way that it repeats its path at regular intervals of time.The to-and-fro motion of an object from its mean position is defined as oscillatory motion.Simple Harmonic Motion is the motion of an object, around a mean position in which the acceleration of the particle is directly proportional to the displacement of the particle.Some Examples of Periodic motion include the motion of hands in a clock, the movement of the Earth around the Sun, and a simple pendulum.Examples of oscillatory motion include a simple pendulum, a vibrating tuning fork, AC current, waves such as sound waves, ocean waves, or light waves, etc.Examples of SHM are the motion ofa spring system i.e., a mass attached to the end of a spring, swings, and the motion of a cantilever. Periodic motion is not always harmonic.Oscillatory Motion is not always periodic.SHM is an example of Periodic and Oscillatory motion. Read More, FrequencyAngular VelocityAngular MomentumSample Questions on Simple Harmonic MotionQuestion 1: Why is Harmonic Motion Periodic? Solution: The sine wave can represent a harmonic motion. When a spring is stretched from its mean position, it oscillates to and fro about the mean position under the influence of a restoring force that is always directed towards the mean and whose magnitude at any instant is proportional to the body's displacement from the mean position at that instant. When there is no friction, the motion tends to be periodic. The harmonic motion is periodic in this case. Question 2: What are Periodic and Non-Periodic Changes? Solution: Periodic changes are those that occur at regular intervals of time, such as the occurrence of day and night, or the change of periods in your school. Non-periodic changes are those that do not occur on a regular basis, such as the freezing of ice to water. Question 3: What is the period of the Earth's revolution around the sun and around its polar axis? what is the motion Earth performs explain? Solution: The earth's revolution around the sun takes one year, and its revolution around its polar axis takes one day. The motion of earth is periodic because after some interval of time it repeats its path. Question 4: What is the frequency of SHM? What two time periods and frequency are related? Solution: The frequency of SHM is the number of oscillations performed by a particle per unit of time. Hertz, or r.p.s. (rotations per second), is the SI unit of frequency. Frequency and time period are related as: Frequency, (f) = 1/ Time period (T) Question 5: A spring with a string constant of 1200 N m−1 is mounted on a horizontal table. A 3 kg mass is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 m before being released. Determine the following: The oscillation's frequency of oscillations,Maximum acceleration of the mass, andThe maximum speed of the mass.Solution: Given: Spring Constant, k = 1200 N/m.Mass of Object, m = 3 kg.Displacement, a = 2 m.(1) Frequency of Oscillation: We know that frequency (f) = 1/Time period (T) T = 20 m/s. Question 6: A mass of 2 kg is attached to the end of spring with a spring constant of 50 N/m. What is the period of the resulting simple harmonic motion? (π = 3.14) Solution: Formula for time period is T = 2π√(m/k) where, m is the mass is the spring constantThus, T = 2π√(50/2) ⇒ T = 2π/√(25) ⇒ T = 2π/5 ⇒ T = 1.26 s So, the time period of the SHM is approximately 1.26 s. Question 7: A block of mass 0.5 kg is attached to a spring (k = 100 N/m). If the block is displaced 0.1 m from its equilibrium position then what is the maximum speed of the block during its motion? Solution: The maximum speed is given by: vmax = Aω where, A is Amplitude of Motionω is Angular Frequency Also, angular frequency ω is given by: ω = √(k/m) where, k is the spring constantGiven: Amplitude(A) = 0.1 mk = 100 N/mm = 0.5 Kg ⇒ vmax = 0.1 × √(10/0.5) ⇒ vmax = 0.1 × √(1000/5) ⇒ vmax = 0.1 × √(200) ⇒ vmax = √2 So, the maximum speed of the block during its motion is √2 m/s. SHM JEE Mains Questions1. A damped harmonic oscillator has a frequency of 5 oscillations per second. For every 10 oscillations, the amplitude of the oscillator drops to half. Find the time taken to drop the amplitude to 1/1000 of the original value. 2. If the length of a simple pendulum is increased by 21% then what is the percentage increase in the time period of the pendulum of the increased length? 3. It is given that the ratio of maximum acceleration to maximum velocity in a SHM is 10 second-1 and at t = 0, the displacement is 5 m. What is the maximum acceleration? Given that the initial phase is π/4 4. If a child is swinging in a sitting position and then he stands up, then how the time period of the swing will be affected. 5. The displacement of a particle in simple harmonic motion is given by x(t) = Asin(πt/90). Find the ratio of kinetic energy to the potential energy at t = 210 seconds A Simple Harmonic Motion, or SHM, is defined as a motion in which the restoring force is directly proportional to the displacement of the body from its mean position. The direction of this restoring force is always towards the mean position. The acceleration of a particle executing simple harmonic motion is given by a(t) = -ω2 x(t). Here, ω is the angular velocity of the particle. Download Complete Chapter Notes of Simple Harmonic Motion Download Now Table of Contents Simple Harmonic, Periodic and Oscillation Motion Simple harmonic motion can be described as an oscillatory motion in which the acceleration of the particle at any position is directly proportional to the displacement from the mean position. It is a special case of oscillatory motion. All the Simple Harmonic Motions are oscillatory and also periodic, but not all oscillatory motions are SHM. Oscillatory motion is also called the harmonic motion of all the oscillatory motions, wherein the most important one is Simple Harmonic Motion (SHM). In this type of oscillatory motion, displacement, velocity and acceleration, and force vary (w.r.t time) in a way that can be described by either sine (or) the cosine functions collectively called sinusoids. Also Read: Simple Pendulum Concepts Spring-Mass System The study of Simple Harmonic Motion is very useful and forms an important tool in understanding the characteristics of sound waves, light waves and alternating currents. Any oscillatory motion which is not simple harmonic can be expressed as a superposition of several harmonic motions of different frequencies. Difference between Periodic, Oscillation and Simple Harmonic Motion Periodic Motion A motion repeats itself after an equal interval of time. For example, uniform circular motion. There is no equilibrium position. There is no restoring force. There is no stable equilibrium position. Oscillation Motion To and fro motion of a particle about a mean position is called an oscillatory motion in which a particle moves on either side of the equilibrium (or) mean position is an oscillatory motion. It is a kind of periodic motion bounded between two extreme points. For example, the oscillation of a simple pendulum, spring-mass system. The object will keep on moving between two extreme points about a fixed point is called the mean position (or) equilibrium position along any path (the path is not a constraint). There will be a restoring force directed towards the equilibrium position (or) mean position. In an oscillatory motion, the net force on the particle is zero at the mean position. The mean position is a stable equilibrium position. Simple Harmonic Motion or SHM It is a special case of oscillation, along with a straight line between the two extreme points (the path of SHM is a constraint). The path of the object needs to be a straight line. There will be a restoring force directed towards the equilibrium position (or) mean position. The mean position in Simple Harmonic Motion is a stable equilibrium. Conditions for SHM: \(\begin{array}{l}\begin{matrix} \overrightarrow{F}\propto -\overrightarrow{x} \\ \overrightarrow{a}\,\propto -\overrightarrow{x} \\ \end{matrix}\end{array} \) Types of Simple Harmonic Motion The SHM, or Simple Harmonic Motion, can be classified into two types: Linear Simple Harmonic Motion When a particle moves to and fro about a fixed point (called equilibrium position) along with a straight line, then its motion is called linear Simple Harmonic Motion. For example, the spring-mass system. Conditions for Linear SHM The restoring force or acceleration acting on the particle should always be proportional to the displacement of the particle and directed towards the equilibrium position. \(\begin{array}{l}\begin{matrix} \overrightarrow{F}\propto -\overrightarrow{x} \\ \overrightarrow{a}\,\propto -\overrightarrow{x} \\ \end{matrix}\end{array} \) \(\begin{array}{l}\overrightarrow{x}-\text{Displacement of particle from equilibrium position}\end{array} \) \(\begin{array}{l}\overrightarrow{F}- \text{Restoring force}\end{array} \) \(\begin{array}{l}\overrightarrow{a}- \text{acceleration}\end{array} \) Angular Simple Harmonic Motion When a system oscillates angular long with respect to a fixed axis, then its motion is called angular simple harmonic motion. Conditions to Execute Angular SHM The restoring torque (or) angular acceleration acting on the particle should always be proportional to the angular displacement of the particle and directed towards the equilibrium position. Τ ∝ θ or α ∝ θ Where, Τ – Torque α – Angular acceleration θ – Angular displacement Simple Harmonic Motion Key Terms Mean Position The point at which net force acting on the particle is zero. From the mean position, the force acting on the particle is \(\begin{array}{l}\overrightarrow{F}\propto -\overrightarrow{x}\end{array} \) \(\begin{array}{l}\overrightarrow{a}\,\propto -\overrightarrow{x}\end{array} \) \(\begin{array}{l}\overrightarrow{{{F}_{net}}}=0\end{array} \) \(\begin{array}{l}\Rightarrow \overrightarrow{a}=0\end{array} \) The force acting on the particle is negative of the displacement. So, this point of equilibrium will be a stable equilibrium. Amplitude in SHM It is the maximum displacement of the particle from the mean position. Time Period and Frequency of SHM The minimum time after which the particle keeps on repeating its motion is known as the time period, or the shortest time taken to complete one oscillation is also defined as the time period. T = 2π/ω Frequency: The number of oscillations per second is defined as the frequency. Frequency = 1/T and, angular frequency ω = 2πf = 2π/T Phase in SHM The phase of a vibrating particle at any instant is the state of the vibrating or oscillating particle regarding its displacement and direction of vibration at that particular instant. The expression and position of a particle as a function of time. x = A sin (ωt + Φ) Where (ωt + Φ) is the phase of the particle, the phase angle at time t = 0 is known as the initial phase. Phase Difference The difference in total phase angles of two particles executing simple harmonic motion with respect to the mean position is known as the phase difference. Two vibrating particles are said to be in the same phase, the phase difference between them is an even multiple of π. ΔΦ = nπ where n = 0, 1, 2, 3, ... Two vibrating particles are said to be in opposite phase if the phase difference between them is an odd multiple of π. ΔΦ = (2n + 1) π where n = 0, 1, 2, 3, ... Simple Harmonic Motion Equation and Its Solution Consider a particle of mass (m) executing Simple Harmonic Motion along a path x o x; the mean position at O. Let the speed of the particle be v0 when it is at position p (at a distance no from O). At t = 0, the particle at P (moving towards the right) At t = t, the particle is at Q (at a distance x from O) With a velocity (v). \(\begin{array}{l}\text{The restoring force}\ \overrightarrow{F}\ \text{at Q is given by}\end{array} \) \(\begin{array}{l}\Rightarrow \overrightarrow{F}=-K\overrightarrow{x}\end{array} \) \ K – is a positive constant \(\begin{array}{l}\Rightarrow \overrightarrow{F}=m\overrightarrow{a}\end{array} \) Here, \(\begin{array}{l}\overrightarrow{a}-\text{acceleration at Q}\end{array} \) \(\begin{array}{l}\Rightarrow m\overrightarrow{a}=-K\overrightarrow{x}\end{array} \) \(\begin{array}{l}\Rightarrow \overrightarrow{a}=-\left(\frac{K}{m} \right)\overrightarrow{x}\end{array} \) \(\begin{array}{l}Put\ \frac{K}{m}={{\omega }^{2}}\end{array} \) \(\begin{array}{l}\Rightarrow \omega =\sqrt{\frac{K}{m}}\end{array} \) \(\begin{array}{l}\Rightarrow \overrightarrow{a}=-\left(\frac{K}{m} \right)\overrightarrow{x}\end{array} \) \(\begin{array}{l}\overrightarrow{a}=-{{\omega }^{2}}\overrightarrow{x}\end{array} \) Since, \(\begin{array}{l}\overrightarrow{a}=\frac{{{d}^{2}}x}{d{{t}^{2}}}\end{array} \) \(\begin{array}{l}\Rightarrow \frac{{{d}^{2}}\overrightarrow{x}}{d{{t}^{2}}}=-{{\omega }^{2}}\overrightarrow{x}\end{array} \) d2x/dt2 + ω2x = 0 This is the differential equation for linear Simple Harmonic Motion. Solutions of Differential Equations of SHM The differential equation for simple harmonic motion has the following solutions: \(\begin{array}{l}x=A\sin \omega t\end{array} \) (This solution when the particle is in its mean position point (O) in figure (a) \(\begin{array}{l}{{x}_{0}}=A\sin \phi\end{array} \) (When the particle is at the position (b) not at mean position) in figure (b) \(\begin{array}{l}x=A\sin \left(\omega t+\phi \right)\end{array} \) (When the particle is at Q at figure (b) any time t). These solutions can be verified by substituting these values in the above differential equation for the linear simple harmonic motion. Angular Simple Harmonic Motion A body free to rotate about an axis can make angular oscillations. For example, a photo frame or a calendar suspended from a nail on the wall. If it is slightly pushed from its mean position and released, it makes angular oscillations. Conditions for an Angular Oscillation to be Angular SHM The body must experience a net torque that is restored in nature. If the angle of oscillation is small, this restoring torque will be directly proportional to the angular displacement. T ∝ -θ T = – kθ T = Iα α = – kθ \(\begin{array}{l}I\frac{{{d}^{2}}\theta }{d{{t}^{2}}}=-K\theta \end{array} \) \(\begin{array}{l}\frac{{{d}^{2}}\theta }{d{{t}^{2}}}=-\left(\frac{K}{I} \right)\theta =-{{\omega _{0}^{2}}}\theta \end{array} \) \(\begin{array}{l}\frac{{{d}^{2}}\theta }{d{{t}^{2}}}+{{\omega }^{2}}\theta =0\end{array} \) The solution of this equation is the angular position of the particle with respect to time. \(\begin{array}{l}\theta =\left({{\theta }_{0}} \right)\sin \left(\omega t+\phi \right)\end{array} \) Then the angular velocity, \(\begin{array}{l}\omega =\left({{\theta }_{0}} \right)\omega \cos \left(\omega t+\phi \right)\end{array} \) θ0 – Amplitude of the angular SHM Example: Simple pendulum Seconds pendulum Torsional pendulum Quantitative Analysis of SHM Let us consider a particle executing Simple Harmonic Motion between A and A1 about passing through the mean position (or) equilibrium position (O). Its analysis is as follows SHM about Position O Displacement x = -A x = 0 x = +A Acceleration |a| = Max a = 0 |a| = Max speed |v| = 0 |v| = Max |v| = 0 Kinetic energy KE = 0 KE = Max KE = 0 Potential energy PE = Max PE = Min PE = Max Equation of Position of a Particle as a Function of Time Let us consider a particle, which is executing SHM at time t = 0, and the particle is at a distance from equilibrium position. Necessary Conditions for Simple Harmonic Motion \(\begin{array}{l}\overrightarrow{F}\propto -\overrightarrow{x}\end{array} \) \(\begin{array}{l}\overrightarrow{a}=-{{\omega }^{2}}x\end{array} \) \(\begin{array}{l}\Rightarrow \overrightarrow{a}=v\frac{dv}{dx}=-{{\omega }^{2}}x\end{array} \) \(\begin{array}{l}\int\limits_{0}^{v}{v\,dv}=\int\limits_{0}^{x}{-{{\omega }^{2}}xdx}\end{array} \) \(\begin{array}{l}\frac{{{v}^{2}}}{2}=\frac{-{{\omega }^{2}}{{x}^{2}}}{2}+c.....(i)\end{array} \) Some conditions we know: At point A v = 0 [x = A] the equation (1) becomes \(\begin{array}{l}0=\frac{-{{\omega }^{2}}{{A}^{2}}}{2}+c\end{array} \) Using, v = 0 \(\begin{array}{l}0 =\frac{-{{\omega }^{2}}{{A}^{2}}}{2}+c\end{array} \) \(\begin{array}{l}c =\frac{{{\omega }^{2}}{{A}^{2}}}{2}\end{array} \) Sub the value of C in equation (1) \(\begin{array}{l}\frac{{{v}^{2}}}{2}=\frac{-{{\omega }^{2}}{{x}^{2}}}{2}+\frac{{{\omega }^{2}}{{A}^{2}}}{2}\end{array} \) \(\begin{array}{l}\Rightarrow {{v}^{2}}=-{{\omega }^{2}}{{x}^{2}}+{{\omega }^{2}}{{A}^{2}}\end{array} \) \(\begin{array}{l}\Rightarrow {{v}^{2}}={{\omega }^{2}}\left({{A}^{2}}-{{x}^{2}} \right)\end{array} \) \(\begin{array}{l}v = \sqrt{{{\omega }^{2}}\left({{A}^{2}}-{{x}^{2}} \right)}\end{array} \) \(\begin{array}{l}v=\omega \sqrt{{{A}^{2}}-{{x}^{2}}}.....(2)\end{array} \) Where is the velocity of the particle executing simple harmonic motion from the definition of instantaneous velocity \(\begin{array}{l}v=\frac{dx}{dt}=\omega \sqrt{{{A}^{2}}-{{x}^{2}}}\end{array} \) \(\begin{array}{l}\Rightarrow \int\limits_{{{x}_{0}}}^{x}{\frac{dx}{\sqrt{{{A}^{2}}-{{x}^{2}}}}}=\int\limits_{0}^{t}{\omega dt}\end{array} \) \(\begin{array}{l}\Rightarrow {{\sin }^{-1}}\left(\frac{x}{A} \right)-{{\sin }^{-1}}\left(\frac{{{x}_{0}}}{A} \right)=\omega t\end{array} \) \(\begin{array}{l}\Rightarrow {{\sin }^{-1}}\left(\frac{x}{A} \right)=\omega t+\phi\end{array} \) \ x = A sin (ωt + Φ) . . . . . (3) Equation (3) – Equation of the position of a particle as a function of time. Case 1: If at t = 0 The particle is at x = x0 \(\begin{array}{l}\Rightarrow {{\sin }^{-1}}\left(\frac{{{x}_{0}}}{A} \right)=\phi\end{array} \) Φ is the initial phase of the particle. Case 2: If at t = 0 The particle is at x = 0 \(\begin{array}{l}\Rightarrow {{\sin }^{-1}}\left(\frac{O}{A} \right)=\phi\end{array} \) i.e. Φ = 0 Case 3: If the particle is at one of its extreme positions, x = A at t = 0 \(\begin{array}{l}\Rightarrow {{\sin }^{-1}}\left(\frac{A}{A} \right)=\phi\end{array} \) ⇒ π/2 = Φ So, the value can be anything depending upon the position of the particle at time t = 0. That is why it is called the initial phase of the particle. Now, if we see the equation of the position of the particle with respect to time, x = A sin (ωt + Φ) sin (ωt + Φ) is the periodic function, whose period is T = 2π/ω Which can be anything, since function can be anything Time Period of SHM The coefficient of t is ω, So, the time period T = 2π/ω ω = 2π/T ω = 2πf ω = Angular frequency of SHM From the expression of particle position as a function of time: \(\begin{array}{l}\text{We can find particles, displacement}\ \left(\overrightarrow{x} \right)\text{, velocity}\ \left(\overrightarrow{v} \right)\text{ and acceleration as follows.}\end{array} \) Velocity in SHM Velocity in SHM is given by v = dx/dt, x = A sin (ωt + Φ) \(\begin{array}{l}v = \frac{d}{dt}A\sin \left(\omega t+\phi \right)=\omega A\cos \left(\omega t+\phi \right)\end{array} \) \(\begin{array}{l}v = A\omega \sqrt{1-{{\sin }^{2}}\left(\omega t+\phi \right)}\end{array} \) Since x = A sin ωt \(\begin{array}{l}\frac{x}{A}=\sin \omega t\end{array} \) \(\begin{array}{l}\frac{{{x}^{2}}}{{{A}^{2}}}={{\sin }^{2}}\omega t\end{array} \) \(\begin{array}{l}\Rightarrow v = A\omega \sqrt{1-\frac{{{x}^{2}}}{{{A}^{2}}}}\end{array} \) \(\begin{array}{l}\Rightarrow v = \omega \sqrt{{{A}^{2}}-{{x}^{2}}}\end{array} \) On squaring both sides, \(\begin{array}{l} {{v}^{2}}={{\omega }^{2}}\left({{A}^{2}}-{{x}^{2}} \right)\end{array} \) \(\begin{array}{l}\Rightarrow \frac{{{v}^{2}}}{{{\omega }^{2}}}={{A}^{2}}-{{x}^{2}}\end{array} \) \(\begin{array}{l}\Rightarrow \frac{{{v}^{2}}}{{{\omega }^{2}}{{A}^{2}}}+\frac{{{x}^{2}}}{{{A}^{2}}}=1\end{array} \) \) this is an equation of an ellipse. The curve between displacement and velocity of a particle executing the simple harmonic motion is an ellipse. When ω = 1, then the curve between v and x will be circular. Acceleration in SHM \(\begin{array}{l}\overrightarrow{a}=\frac{dv}{dt}=\frac{d}{dt}\left(A\omega \cos \omega t+\phi \right)\end{array} \) \(\begin{array}{l}\Rightarrow \overrightarrow{a}=-{{\omega }^{2}}A\sin \left(\omega t+\phi \right)\end{array} \) \(\begin{array}{l}\Rightarrow \left| a \right|=-{{\omega }^{2}}x\end{array} \) Hence, the expression for displacement, velocity and acceleration in linear simple harmonic motion is x = A sin (ωt + Φ) \(\begin{array}{l}v = A\omega \cos \left(\omega t+\phi \right)=\omega \sqrt{{{A}^{2}}-{{x}^{2}}}\end{array} \) and \(\begin{array}{l}a = -A{{\omega }^{2}}\sin \left(\omega t+\phi \right)\end{array} \) \(\begin{array}{l}\Rightarrow a=-{{\omega }^{2}}x\end{array} \) Energy in Simple Harmonic Motion (SHM) The system that executes SHM is called the harmonic oscillator. Consider a particle of mass m, executing linear simple harmonic motion with angular frequency (ω) and amplitude (A), \(\begin{array}{l}\text{the displacement}\ \left(\overrightarrow{x} \right)\text{, velocity}\ \left(\overrightarrow{v} \right)\text{and acceleration}\ \left(\overrightarrow{a} \right)\text{at any time t are given by}\end{array} \) x = A sin (ωt + Φ) \(\begin{array}{l}v = A\omega \cos \left(\omega t+\phi \right)=\omega \sqrt{{{A}^{2}}-{{x}^{2}}}\end{array} \) \(\begin{array}{l}a = -{{\omega }^{2}}A\sin \left(\omega t+\phi \right)=-{{\omega }^{2}}x\end{array} \) \(\begin{array}{l}\text{The restoring force}\ \left(\overrightarrow{F} \right)\text{acting on the particle is given by}\end{array} \) F = -kx, where k = mω2. Kinetic Energy of a Particle in SHM Kinetic Energy \(\begin{array}{l}=\frac{1}{2}m{{v}^{2}}\end{array} \) \(\begin{array}{l}=\frac{1}{2}m{{\omega }^{2}}\left({{A}^{2}}-{{x}^{2}} \right)\end{array} \) Since, \(\begin{array}{l}{{v}^{2}}={{\omega }^{2}}{{A}^{2}}{{\cos }^{2}}\left(\omega t+\phi \right)\end{array} \) \(\begin{array}{l}=\frac{1}{2}m{{\omega }^{2}}{{A}^{2}}{{\cos }^{2}}\left(\omega t+\phi \right)\end{array} \) \(\begin{array}{l}=\frac{1}{2}m{{\omega }^{2}}\left({{A}^{2}}-{{x}^{2}} \right)\end{array} \) Therefore, the Kinetic Energy \(\begin{array}{l}=\frac{1}{2}m{{\omega }^{2}}{{A}^{2}}{{\cos }^{2}}\left(\omega t+\phi \right)\end{array} \) \(\begin{array}{l}=\frac{1}{2}m{{\omega }^{2}}\left({{A}^{2}}-{{x}^{2}} \right)\end{array} \) Potential Energy of SHM The total work done by the restoring force in displacing the particle from (x = 0) (to mean position) to x = x: When the particle has been displaced from x to x + dx, the work done by restoring force is dw = F dx = -kx dx \ \(\begin{array}{l}w = \int{dw}=\int\limits_{0}^{x}{-kxdx}=\frac{-k{{x}^{2}}}{2}\end{array} \) \(\begin{array}{l}=-\frac{1}{2}m{{\omega }^{2}}{{x}^{2}}\end{array} \) \(\begin{array}{l}w = -\frac{1}{2}m{{\omega }^{2}}{{A}^{2}}{{\sin }^{2}}\left(\omega t+\phi \right)\end{array} \) Potential Energy = -(work done by restoring force) \(\begin{array}{l}=\frac{1}{2}m{{\omega }^{2}}{{x}^{2}}\end{array} \) \(\begin{array}{l}=\frac{1}{2}m{{\omega }^{2}}{{A}^{2}}{{\sin }^{2}}\left(\omega t+\phi \right)\end{array} \) E = KE + PE \(\begin{array}{l}E=\frac{1}{2}m{{\omega }^{2}}\left({{A}^{2}}-{{x}^{2}} \right)+\frac{1}{2}m{{\omega }^{2}}{{x}^{2}}\end{array} \) \(\begin{array}{l}E =\frac{1}{2}m{{\omega }^{2}}{{A}^{2}}\end{array} \) Hence, the particle's total energy in SHM is constant, independent of the instantaneous displacement. ⇒ Relationship between kinetic energy, potential energy and time in Simple Harmonic Motion at t = 0, when x = ±A. ⇒ Variation of kinetic energy and potential energy in Simple Harmonic Motion with displacement. Geometrical Interpretation of Simple Harmonic Motion If a particle is moving with uniform speed along the circumference of a circle, then the straight line motion of the foot of the perpendicular drawn from the particle on the diameter of the circle is called simple harmonic motion. SHM as a Projection of Circular Motion The particle is at position P at t = 0 and revolves along a circle with a constant angular velocity (ω). The projection of P on the diameter along the x-axis (M). At the later time (t), the particle is at Q. Now, its projection on the diameter along the x-axis is N. As the particle P revolves around in a circle anti-clockwise, its projection M follows it up, moving back and forth along the diameter, such that the displacement of the point of projection at any time (t) is the x-component of the radius vector (A). x = A cos (ωt + Φ) . . . . . . (1) y = A sin (ωt + Φ) . . . . . (2) Thus, we see that the uniform circular motion is the combination of two mutually perpendicular linear harmonic oscillations. It implies that P is under uniform circular motion, (M and N) and (K and L) are performing simple harmonic motion about O with the same angular speed ω as that of P. P is under uniform circular motion, which will have centripetal acceleration along A (radius vector). \(\begin{array}{l}\overrightarrow{{{a}_{c}}}=\left({{\omega }^{2}}A \right)\end{array} \) (towards the centre) It can be resolved into two components: \(\begin{array}{l}{{a}_{c}}=\left({{\omega }^{2}}A \right){{\sin }^{2}}\left(\omega t+\phi \right)\end{array} \) \(\begin{array}{l}{{a}_{L}}=\left({{\omega }^{2}}A \right)\cos \left(\omega t+\phi \right)\end{array} \) aN and aL acceleration correspond to the points N and L, respectively. In the above discussion, the foot of the projection on the x-axis is called a horizontal phasor. Similarly, the foot of the perpendicular on the y-axis is called the vertical phasor. As we already know the vertical and horizontal phasor will execute the simple harmonic motion of amplitude A and angular frequency ω. The phases of the two SHMs differ by π/2. Video Lessons Simple Harmonic Motion Short Notes Problem-solving Strategy in Horizontal Phasor Let us assume a circle of radius equal to the amplitude of SHM. Assume a particle rotating in a circular path moving with constant same as that of angular frequency of SHM in the clockwise direction. The angle made by the particle at t = 0 with the upper vertical axis is equal to φ (phase constant). The horizontal component of the velocity of a particle gives you the velocity of a particle performing the simple harmonic motion. The component of the acceleration of a particle in the horizontal direction is equal to the acceleration of the particle performing SHM. [In uniform circular acceleration centripetal only ac = ω2A]. Simple harmonic motion is a periodic motion in which acceleration is directly proportional to its displacement and is directed towards the mean position. The restoring force is given by the formula F = -kx The negative sign shows that the force is in the opposite direction. k is the force constant. x is the displacement of the string from the equilibrium position. The harmonic motion is when the restoring force is proportional to the displacement but in opposite directions. Simple Harmonic Motion is a harmonic motion with a constant amplitude and the same frequency. The periodic motion of gradually decreasing amplitude is called the damped simple harmonic motion. Distance travelled by the particle from its mean position at any instant is called the displacement of the particle executing simple harmonic motion. At the mean position, the energy is entirely kinetic energy. At the extreme position, the energy is entirely potential energy. The phase difference between displacement and acceleration of the particle executing simple harmonic motion is π radian. The maximum displacement of the particle is called the amplitude of motion. Put your understanding of this concept to test by answering a few MCQs. Click 'Start Quiz' to begin! Select the correct answer and click on the "Finish" buttonCheck your score and answers at the end of the quiz Visit BYJU'S for all JEE related queries and study materials 0 out of 0 arewrong 0 out of 0 are correct 0 out of 0 are Unattempted View Quiz Answers and Analysis When a guitar string is plucked or a spring moves up and down, the time interval between each oscillation is defined as periodic motion. The time that one oscillation is completed is called the period. In addition to indicating repeated oscillations, a period can also represent one event. The unit of measurement for a period is typically indicated in seconds. Frequency When a period is repeated, the number of oscillations per unit of time is indicated as frequency. Mathematically, the frequency is expressed by the following formula: $f=\frac{1}{T}$ Where, f = frequency T = Time period 1 = Constant The SI unit for frequency is in cycles per second, also known as hertz (Hz). One cycle is equal to one oscillation. Oscillations are repetitive for a number of cycles. Review Questions 1. A _____ causes a disturbance in a system that activates an oscillation. 2. Waves created by oscillations carry energy. T/F 3. The restoring force of an object _____ when the deformation is increased. is increased is decreased stays the same 4. One cycle is equal to how many oscillations? 5. The term ______ refers to the number of oscillations per unit of time. Simple Harmonic Motion: A special Periodic Motion Oscillations are very common in nature and by human made objects because they occur in so many different ways. One type of oscillation is simple harmonic motion. It refers to oscillatory motion that is directly proportional to displacement, and the system in which oscillations occur is called a simple harmonic oscillator. If there is no friction or other nonconservative forces that dampen oscillations, a simple harmonic oscillator will continue to oscillate indefinitely with equal displacement on either side of the equilibrium position. The maximum displacement from equilibrium is called amplitude. Amplitude and displacement of objects such as a metal coiled spring are in meters, whereas sound oscillations are indicated in units of pressure. Simple Harmonic Motion A significant fact about simple harmonic motion is that the period t and frequency f are independent of amplitude. For example, guitar strings will oscillate at the same frequency whether it is plucked gently or hard. For this reason, simple harmonic oscillations are used to operate clocks because the period remains constant. The only factors that affect the period and frequency of simple harmonic motion are mass and the force constant k. Whenever a harmonic oscillator is stiff, a large force k is required to activate it. Also, it will have a smaller time period than an object that is less stiff. The period of a harmonic oscillator is impacted by its mass. The more massive the system is, the longer its period. The Link between Harmonic Motion and Waves All simple harmonic motions are related to sine and cosine waves. The displacement is a function of time in any harmonic motion as oscillations occur with a period T. The velocity of the motion is also a function of time. At maximum displacement from equilibrium, velocity and time are zero. The Simple Pendulum One type of simple harmonic oscillator is a simple pendulum. A simple pendulum is an object that has a small mass, which is suspended by a light wire or string. When a simple pendulum is displaced from equilibrium, it swings in an arc. The length of the displacement is called the arc length and is identified as s. When displacement occurs, a restoring force is created that is in the direction towards the equilibrium position. This restoring force is directly proportional to the displacement. Two factors affect the period of a simple pendulum, which is the time duration at which one oscillation takes place. One factor is the length of the string or wire, and the second factor is the acceleration due to gravity. The period T is nearly independent of amplitude and mass. Fig.1: Simple Pendulum – Harmonic Oscillator Review Questions 6. A simple harmonic motion is never capable of oscillating indefinitely. T/F 7. A significant fact about simple harmonic motion is that ____ is independent of amplitude. the period frequency Both a and b 8. Which factor is true about affecting the period and frequency of simple harmonic motion? The less stiff an object is, the smaller its time period. Whenever a harmonic oscillator is stiff, a larger force is required to activate. The more massive a system is, the longer the period. All of the above 9. When an object oscillates and reaches its maximum displacement, velocity and time are _____. 10. List two factors that affect the time period of a simple pendulum. Energy and the Simple Harmonic Oscillator A simple harmonic oscillator has both potential energy and kinetic energy. When an object is deformed and at the moment it is not moving, it has stored potential energy. Because a simple harmonic oscillator has no dissipative forces, it has kinetic energy. Therefore, as an undamped simple harmonic motion takes place, the energy oscillates back and forth between kinetic and potential energy. An example is the oscillations of a spring. When it is completely compressed and is not moving, all energy is stored as potential energy. When the spring decompresses, the elastic potential energy is converted to kinetic energy. At the equilibrium position, the entire energy of the spring is kinetic. As it moves passed equilibrium, the energy in the spring is converting back to potential energy. Velocity during Oscillations When a simple harmonic oscillation has reached its maximum displacement position, the velocity is zero. In this position, all of the energy is in the potential form and there is no kinetic energy. As the restoring force causes the oscillation to move towards equilibrium, the potential energy decreases, and the kinetic energy increases. Energy is shared by both, but the total energy does not change. When the oscillation reaches the equilibrium position, its velocity is at a maximum level. Maximum velocity depends on three factors: Maximum velocity is directly proportional to amplitude. Maximum velocity is greater for stiffer objects. Maximum velocity is smaller for objects that have larger masses. Uniform Circular Motion When an object moves in a circular path with a constant angular velocity and uniform circular motion, a simple harmonic motion takes place. The motion is back and forth on the x-axis. The period T of an oscillator is the time it takes for the object to make one complete revolution. When viewing a merry-go-round from a distance, any object exhibits simple harmonic motion when it goes back and forth between left and right positions as it turns to create uniform circular motion. Review Questions 16. Give an example of when the damping of an oscillator is desirable. 17. As the oscillations of harmonic motion slow down due to damping, the net force _____. increases decreases stays the same 18. _______ damping refers to a system that is slow and sluggish. 19. When driving an object with a frequency equal to its natural frequency, a condition called ______ occurs. 20. Whenever the damping of a harmonic oscillator becomes smaller, the amplitude of the oscillator also becomes smaller. T/F Review Answers Force T a One Frequency F c d zero Length and acceleration due to gravity c zero T It remains constant A The shocks on an automobile b c resonance F The article discusses the role of transformers in electronic power supply, focusing on how they step voltage up ... Read More This article covers diode testing and troubleshooting of diodes, emphasizing the importance of correct polarity and biasing in ... Read More The article provides an overview of the key specifications of diode, including maximum ratings, operating conditions, and their ... Read More The article discusses the electrical characteristics of PN junction diode, focusing on their forward and reverse bias behavior, ... Read More The article provides an overview of atomic theory, focusing on the structure of matter, atoms, and the periodic ... Read More This article discusses atomic states and the three types of atomic bonding: ionic bonding, covalent bonding, and metallic ... Read More The article explores how Industry 4.0 leverages technologies like RFID, M2M communication, and IIoT for seamless data exchange, ... Read More The article provides an overview of mechatronic system, their components, and functions, highlighting the integration of mechanical, electronic, ... Read More The article discusses the role of mechatronic system in modern society, highlighting their applications in vehicles, robotics, and ... Read More Learn how to use an ohmmeter to test a bipolar junction transistor (BJT), identify its leads, and evaluate ... Read More This article discusses the advantages and disadvantages of ICs. As you will see, when all factors are considered, ... Read More The frequency of simple harmonic motion like a mass on a spring is determined by the mass m and the stiffness of the spring expressed in terms of a spring constant k (see Hooke's Law): The air outside an IB exam room after the final paper feels different. It's a potent mix of exhaustion, relief,... By the end of this section, you will be able to: Define the terms period and frequency List the characteristics of simple harmonic motion Explain the concept of phase shift Write the equations of motion for the system of a mass and spring undergoing simple harmonic motion Describe the motion of a mass oscillating on a vertical spring When you pluck a guitar string, the resulting sound has a steady tone and lasts a long time (Figure 15.2). The string vibrates around an equilibrium position, and one oscillation is completed when the string starts from the initial position, travels to one of the extreme positions, then to the other extreme position, and returns to its initial position. We define periodic motion to be any motion that repeats itself at regular time intervals, such as exhibited by the guitar string or by a child swinging on a swing. In this section, we study the basic characteristics of oscillations and their mathematical description. Figure 15.2 When a guitar string is plucked, the string oscillates up and down in periodic motion. The vibrating string causes the surrounding air molecules to oscillate, producing sound waves. (credit: Yutaka Tsutano) In the absence of friction, the time to complete one oscillation remains constant and is called the period (T). Its units are usually seconds, but may be any convenient unit of time. The word 'period' refers to the time for some event whether repetitive or not, but in this chapter, we shall deal primarily in periodic motion, which is by definition repetitive. A concept closely related to period is the frequency of an event. Frequency (f) is defined to be the number of events per unit time. For periodic motion, frequency is the number of oscillations per unit time. The relationship between frequency and period is The SI unit for frequency is the hertz (Hz) and is defined as one cycle per second: 1Hz=1cyclesor1Hz=1s=1s−1.1Hz=1cyclesor1Hz=1s=1s−1.A cycle is one complete oscillation. Determining the Frequency of Medical Ultrasound Ultrasound machines are used by medical professionals to make images for examining internal organs of the body. An ultrasound machine emits high-frequency sound waves, which reflect off the organs, and a computer receives the waves, using them to create a picture. We can use the formulas presented in this module to determine the frequency, based on what we know about oscillations. Consider a medical imaging device that produces ultrasound by oscillating with a period of 0.400µs0.400µs. What is the frequency of this oscillation? Strategy The period (T) is given and we are asked to find frequency (f). Solution Substitute 0.400µs0.400µs for T in f=1Tf=1T: f=1T=10.400×10−6s.f=1T=10.400×10−6s. Solve to find f=2.50×106Hz.f=2.50×106Hz. Significance This frequency of sound is much higher than the highest frequency that humans can hear (the range of human hearing is 20 Hz to 20,000 Hz); therefore, it is called ultrasound. Appropriate oscillations at this frequency generate ultrasound used for noninvasive medical diagnoses, such as observations of a fetus in the womb. A very common type of periodic motion is called simple harmonic motion (SHM). A system that oscillates with SHM is called a simple harmonic oscillator. In simple harmonic motion, the acceleration of the system, and therefore the net force, is proportional to the displacement and acts in the opposite direction of the displacement. A good example of SHM is an object with mass m attached to a spring on a frictionless surface, as shown in Figure 15.3. The object oscillates around the equilibrium position, and the net force on the object is equal to the force provided by the spring. This force obeys Hooke's law Fs=−kx,Fs=−kx, as discussed in a previous chapter. If the net force can be described by Hooke's law and there is no damping (slowing down due to friction or other nonconservative forces), then a simple harmonic oscillator oscillates with equal displacement on either side of the equilibrium position, as shown for an object on a spring in Figure 15.3. The maximum displacement from equilibrium is called the amplitude (A). The units for amplitude and displacement are the same but depend on the type of oscillation. For the object on the spring, the units of amplitude and displacement are meters. Figure 15.3 An object attached to a spring sliding on a frictionless surface is an uncomplicated simple harmonic oscillator. In the above set of figures, a mass is attached to a spring and placed on a frictionless table. The other end of the spring is attached to the wall. The position of the mass, when the spring is neither stretched nor compressed, is marked as x=0x=0 and is the equilibrium position. (a) The mass is displaced to a position x=Ax=A and released from rest. (b) The mass accelerates as it moves in the negative x-direction, reaching a maximum negative velocity at x=0x=0. (c) The mass continues to move in the negative x-direction, slowing until it comes to a stop at x=−Ax=−A. (d) The mass now begins to accelerate in the positive x-direction, reaching a positive maximum velocity at x=0x=0. (e) The mass then continues to move in the positive direction until it stops at x=Ax=A. The mass continues in SHM that has an amplitude A and a period T. The object's maximum speed occurs as it passes through equilibrium. The stiffer the spring is, the smaller the period T. The greater the mass of the object is, the greater the period T. What is so significant about SHM? For one thing, the period T and frequency f of a simple harmonic oscillator are independent of amplitude. The string of a guitar, for example, oscillates with the same frequency whether plucked gently or hard. Two important factors do affect the period of a simple harmonic oscillator. The period is related to how stiff the system is. A very stiff object has a large force constant (k), which causes the system to have a smaller period. For example, you can adjust a diving board's stiffness—the stiffer it is, the faster it vibrates, and the shorter its period. Period also depends on the mass of the oscillating system. The more massive the system is, the longer the period. For example, a heavy person on a diving board bounces up and down more slowly than a light one. In fact, the mass m and the force constant k are the only factors that affect the period and frequency of SHM. To derive an equation for the period and the frequency, we must first define and analyze the equations of motion. Note that the force constant is sometimes referred to as the spring constant. Consider a block attached to a spring on a frictionless table (Figure 15.4). The equilibrium position (the position where the spring is neither stretched nor compressed) is marked as x=0x=0. At the equilibrium position, the net force is zero. Figure 15.4 A block is attached to a spring and placed on a frictionless table. The equilibrium position, where the spring is neither extended nor compressed, is marked as x=0.x=0. Work is done on the block to pull it out to a position of x=+A,x=+A, and it is then released from rest. The maximum x-position (A) is called the amplitude of the motion. The block begins to oscillate in SHM between x=+Ax=+A and x=−A,x=−A, where A is the amplitude of the motion and T is the period of the oscillation. The period is the time for one oscillation. Figure 15.5 shows the motion of the block as it completes one and a half oscillations after release. Figure 15.6 shows a plot of the position of the block versus time. When the position is plotted versus time, it is clear that the data can be modeled by a cosine function with an amplitude A and a period T. The cosine function cosθcosθ repeats every multiple of 2π,2π, whereas the motion of the block repeats every period T. However, the function cos(2πTt)cos(2πTt) repeats every integer multiple of the period. The maximum of the cosine function is one, so it is necessary to multiply the cosine function by the amplitude A. x(t)=Acos(2πTt)=Acos(ωt).x(t)=Acos(2πTt)=Acos(ωt). Recall from the chapter on rotation that the angular frequency equals ω=dθdtω=dθdt. In this case, the period is constant, so the angular frequency is defined as 2π2π divided by the period, ω=2πTω=2πT. Figure 15.5 A block is attached to one end of a spring and placed on a frictionless table. The other end of the spring is anchored to the wall. The equilibrium position, where the net force equals zero, is marked as x=0m.x=0m. Work is done on the block, pulling it out to x=+Ax=+A, and the block is released from rest. The block oscillates between x=+Ax=+A and x=−Ax=−A. The force is also shown as a vector. Figure 15.6 A graph of the position of the block shown in Figure 15.5 as a function of time. The position can be modeled as a periodic function, such as a cosine or sine function. The equation for the position as a function of time x(t)=Acos(ωt)x(t)=Acos(ωt) is good for modeling data, where the position of the block at the initial time t=0.00st=0.00s is at the amplitude A and the initial velocity is zero. Often when taking experimental data, the position of the mass at the initial time t=0.00st=0.00s is not equal to the amplitude and the initial velocity is not zero. Consider 10 seconds of data collected by a student in lab, shown in Figure 15.7. Figure 15.7 Data collected by a student in lab indicate the position of a block attached to a spring, measured with a sonic range finder. The data are collected starting at time t=0.00s,t=0.00s, but the initial position is near position x≈−0.80cm≠3.00cmx≈−0.80cm≠3.00cm, so the initial position does not equal the amplitude x0=+Ax0=+A. The velocity is the time derivative of the position, which is the slope at a point on the graph of position versus time. The velocity is not v=0.00m/sv=0.00m/s at time t=0.00st=0.00s, as evident by the slope of the graph of position versus time, which is not zero at the initial time. The data in Figure 15.7 can still be modeled with a periodic function, like a cosine function, but the function is shifted to the right. This shift is known as a phase shift and is usually represented by the Greek letter phi (ϕ)(ϕ). The equation of the position as a function of time for a block on a spring becomes x(t)=Acos(ωt+ϕ).x(t)=Acos(ωt+ϕ). This is the generalized equation for SHM where t is the time measured in seconds, ωω is the angular frequency with units of inverse seconds, A is the amplitude measured in meters or centimeters, and ϕϕ is the phase shift measured in radians (Figure 15.8). It should be noted that because sine and cosine functions differ only by a phase shift, this motion could be modeled using either the cosine or sine function. Figure 15.8 (a) A cosine function. (b) A cosine function shifted to the left by an angle ϕϕ. The angle ϕϕ is known as the phase shift of the function. The velocity of the mass on a spring, oscillating in SHM, can be found by taking the derivative of the position equation: v(t)=dxdt=ddt(Acos(ωt+ϕ))=−Aωsin(ωt+ϕ)=−vmaxsin(ωt+ϕ).v(t)=dxdt=ddt(Acos(ωt+ϕ))=−Aωsin(ωt+ϕ)=−vmaxsin(ωt+ϕ). Because the sine function oscillates between –1 and +1, the maximum velocity is the amplitude times the angular frequency, vmax=Aωvmax=Aω. The maximum velocity occurs at the equilibrium position (x=0)(x=0) when the mass is moving toward x=+Ax=+A. The maximum velocity in the negative direction is attained at the equilibrium position (x=0)(x=0) when the mass is moving toward x=−Ax=−A and is equal to −vmax−vmax. The acceleration of the mass on the spring can be found by taking the time derivative of the velocity: a(t)=dvdt=ddt(−Aωsin(ωt+ϕ))=−Aω2cos(ωt+φ)=−amaxcos(ωt+ϕ).a(t)=dvdt=ddt(−Aωsin(ωt+ϕ))=−Aω2cos(ωt+φ)=−amaxcos(ωt+ϕ). The maximum acceleration is amax=Aω2amax=Aω2. The maximum acceleration occurs at the position (x=−A)(x=−A), and the acceleration at the position (x=−A)(x=−A) and is equal to amaxamax. In summary, the oscillatory motion of a block on a spring can be modeled with the following equations of motion: x(t)=Acos(ωt+ϕ)x(t)=Acos(ωt+ϕ) v(t)=−vmaxsin(ωt+ϕ)v(t)=−vmaxsin(ωt+ϕ) a(t)=−amaxcos(ωt+ϕ)a(t)=−amaxcos(ωt+ϕ) Here, A is the amplitude of the motion, T is the period, ϕϕ is the phase shift, and ω=2πT=2πfω=2πT=2πf is the angular frequency of the motion of the block. Determining the Equations of Motion for a Block and a Spring A 2.00-kg block is placed on a frictionless surface. A spring with a force constant of k=32.00N/mk=32.00N/m is attached to the block, and the opposite end of the spring is attached to the wall. The spring can be compressed or extended. The equilibrium position is marked as x=0.00m.x=0.00m. Work is done on the block, pulling it out to x=+0.02m.x=+0.02m. The block is released from rest and oscillates between x=+0.02mx=+0.02m and x=−0.02m.x=−0.02m. The period of the motion is 1.57 s. Determine the equations of motion. Strategy We first find the angular frequency. The phase shift is zero, ϕ=0.00rad,ϕ=0.00rad, because the block is released from rest at x=A=+0.02m.x=A=+0.02m. Once the angular frequency is found, we can determine the maximum velocity and maximum acceleration. Solution The angular frequency can be found and used to find the maximum velocity and maximum acceleration: ω=2π1.57s=4.00s−1;vmax=Aω=0.02m(4.00s−1)=0.08m/s; amax=Aω2=0.02m(4.00s−1)2=0.32m/s2.ω=2π1.57s=4.00s−1;vmax=Aω=0.02m(4.00s−1)=0.08m/s; amax=Aω2=0.02m(4.00s−1)2=0.32m/s2. All that is left is to fill in the equations of motion: x(t)=Acos(ωt+ϕ)=(0.02m)cos(4.00s−1t); v(t)=−vmaxsin(ωt+ϕ)=(−0.08m/s)sin(4.00s−1t); a(t)=−amaxcos(ωt+ϕ)=(−0.32m/s2)cos(4.00s−1t).x(t)=Acos(ωt+ϕ)=(0.02m)cos(4.00s−1t); v(t)=−vmaxsin(ωt+ϕ)=(−0.08m/s)sin(4.00s−1t); a(t)=−amaxcos(ωt+ϕ)=(−0.32m/s2)cos(4.00s−1t). Significance The position, velocity, and acceleration can be found for any time. It is important to remember that when using these equations, your calculator must be in radians mode. One interesting characteristic of the SHM of an object attached to a spring is that the angular frequency, and therefore the period and frequency of the motion, depend on only the mass and the force constant, and not on other factors such as the amplitude of the motion. We can use the equations of motion and Newton's second law (F→net=ma→)(F→net=ma→) to find equations for the angular frequency, frequency, and period. Consider the block on a spring on a frictionless surface. There are three forces on the mass: the weight, the normal force, and the force due to the spring. The only two forces that act perpendicular to the surface are the weight and the normal force, which have equal magnitudes and opposite directions, and thus sum to zero. The only force that acts parallel to the surface is the force due to the spring, so the net force must be equal to the force of the spring: Fx=−kx; ma=−kx; md2xdt2=−kx; d2xdt2=−kmx.Fx=−kx; ma=−kx; md2xdt2=−kx; d2xdt2=−kmx. Substituting the equations of motion for x and a gives us −Aω2cos(ωt+ϕ)=−kmAcos(ωt+ϕ).−Aω2cos(ωt+ϕ)=−kmAcos(ωt+ϕ). Cancelling out like terms and solving for the angular frequency yields The angular frequency depends only on the force constant and the mass, and not the amplitude. The angular frequency is defined as ω=2π/T,ω=2π/T, which yields an equation for the period of the motion: The period also depends only on the mass and the force constant. The greater the mass, the longer the period. The stiffer the spring, the shorter the period. The frequency is f=1T=12πkm.f=1T=12πkm. When a spring is hung vertically and a block is attached and set in motion, the block oscillates in SHM. In this case, there is no normal force, and the net effect of the force of gravity is to change the equilibrium position. Consider Figure 15.9. Two forces act on the block: the weight and the force of the spring. The weight is constant and the force of the spring changes as the length of the spring changes. Figure 15.9 A spring is hung from the ceiling. When a block is attached, the block is at the equilibrium position where the weight of the block is equal to the force of the spring. (a) The spring is hung from the ceiling and the equilibrium position is marked as yoyo. (b) A mass is attached to the spring and a new equilibrium position is reached (y1=yo−Δyy1=yo−Δy) when the force provided by the spring equals the weight of the mass. (c) The free-body diagram of the mass shows the two forces acting on the mass: the weight and the force of the spring. When the block reaches the equilibrium position, as seen in Figure 15.9, the force of the spring equals the weight of the block, Fnet=Fs−mg=0Fnet=Fs−mg=0, where −k(−Δy)=mg.−k(−Δy)=mg. From the figure, the change in the position is Δy=y0−y1Δy=y0−y1 and since −k(−Δy)=mg−k(−Δy)=mg, we havek(y0−y1)−mg=0.k(y0−y1)−mg=0. If the block is displaced and released, it will oscillate around the new equilibrium position. As shown in Figure 15.10, if the position of the block is recorded as a function of time, the recording is a periodic function. If the block is displaced to a position y, the net force becomes Fnet=k(y0−y)−mg=0Fnet=k(y0−y)−mg=0. But we found that at the equilibrium position, mg=kΔy=ky0−ky1mg=kΔy=ky0−ky1. Substituting for the weight in the equation yields Fnet=ky0−ky−(ky0−ky1)=−k(y−y1).Fnet=ky0−ky−(ky0−ky1)=−k(y−y1). Recall that y1y1 is just the equilibrium position and any position can be set to be the point y=0.00m.y=0.00m. So let's set y1y1 to y=0.00m.y=0.00m. The net force then becomes Fnet=−ky; md2ydt2=−ky.Fnet=−ky; md2ydt2=−ky. This is just what we found previously for a horizontally sliding mass on a spring. The constant force of gravity only served to shift the equilibrium location of the mass. Therefore, the solution should be the same form as for a block on a horizontal spring, y(t)=Acos(ωt+ϕ).y(t)=Acos(ωt+ϕ). The equations for the velocity and the acceleration also have the same form as for the horizontal case. Note that the inclusion of the phase shift means that the motion can actually be modeled using either a cosine or a sine function, since these two functions only differ by a phase shift. Figure 15.10 Graphs of y(t), v(t), and a(t) versus t for the motion of an object on a vertical spring. The net force on the object can be described by Hooke's law, so the object undergoes SHM. Note that the initial position has the vertical displacement at its maximum value A; v is initially zero and then negative as the object moves down; the initial acceleration is negative, back toward the equilibrium position and becomes zero at that point. To-and-fro periodic motion in science and engineering Simple harmonic motion shown both in real space and phase space. The orbit is periodic. (Here the velocity and position axes have been reversed from the standard convention to align the two diagrams) Part of a series onClassical mechanics F = d p d t {\displaystyle {\textbf {F}}={\frac {d\mathbf {p} }{dt}}} Second law of motion History Timeline Textbooks Branches Applied Celestial Continuum Dynamics Field theory Kinematics Kinetics Statics Statistical mechanics Fundamentals Acceleration Angular momentum Couple D'Alembert's principle Energy kinetic potential Force Frame of reference Inertial frame of reference Impulse Inertia / Moment of inertia Mass Mechanical power Mechanical work Moment Momentum Space Speed Time Torque Velocity Virtual work Formulations Newton's laws of motion Analytical mechanics Lagrangian mechanicsHamiltonian mechanicsRouthian mechanicsHamilton–Jacobi equationAppell's equation of motionKoopman–von Neumann mechanics Core topics Damping Displacement Equations of motion Euler's laws of motion Fictitious force Friction Harmonic oscillator Inertial / Non-inertial reference frame Motion (linear) Newton's law of universal gravitation Newton's laws of motion Relative velocity Rigid body dynamics Euler's equations Simple harmonic motion Vibration Rotation Circular motion Rotating reference frame

Centripetal force Centrifugal force reactive Coriolis force Pendulum Tangential speed Rotational frequency Angular acceleration / displacement / frequency / velocity Scientists Kepler Galileo Huygens Newton Horrocks Halley Maupertuis Daniel Bernoulli Johann Bernoulli Euler d'Alembert Clairaut Lagrange Laplace Poisson Hamilton Jacobi Cauchy Routh Liouville Appell Gibbs Koopman von Neumann Physics portal Categoryvte In mechanics and physics, simple harmonic motion (sometimes abbreviated as SHM) is a special type of periodic motion an object experiences by means of a restoring force whose magnitude is directly proportional to the distance of the object from an equilibrium position and acts towards the equilibrium position. It results in an oscillation that is described by a sinusoid which continues indefinitely (if uninhibited by friction or any other dissipation of energy).[1] Simple harmonic motion can serve as a mathematical model for a variety of motions, but is typified by the oscillation of a mass on a spring when it is subject to the linear elastic restoring force given by Hooke's law. The motion is sinusoidal in time and demonstrates a single resonant frequency. Other phenomena can be modeled by simple harmonic motion, including the motion of a simple pendulum, although for it to be an accurate model, the net force on the object at the end of the pendulum must be proportional to the displacement (and even so, it is only a good approximation when the angle of the swing is small; see small-angle approximation). Simple harmonic motion can also be used to model molecular vibration. A mass-spring system is a classic example of simple harmonic motion. Simple harmonic motion provides a basis for the characterization of more complicated periodic motion through the techniques of Fourier analysis. The motion of a particle moving along a straight line with an acceleration whose direction is always toward a fixed point on the line and whose magnitude is proportional to the displacement from the fixed point is called simple harmonic motion.[2] In the diagram, a simple harmonic oscillator, consisting of a weight attached to one end of a spring, is shown. The other end of the spring is connected to a rigid support such as a wall. If the system is left at rest at the equilibrium position then there is no net force acting on the mass. However, if the mass is displaced from the equilibrium position, the spring exerts a restoring elastic force that obeys Hooke's law. Mathematically, F = − k x , {\displaystyle \mathbf {F} =-k\mathbf {x} ,} where F is the restoring elastic force exerted by the spring (in SI units: N), k is the spring constant (N·m−1), and x is the displacement from the equilibrium position (in metres). For any simple mechanical harmonic oscillator: When the system is displaced from its equilibrium position, a restoring force that obeys Hooke's law tends to restore the system to equilibrium. Once the mass is displaced from its equilibrium position, it experiences a net restoring force. As a result, it accelerates and starts going back to the equilibrium position. When the mass moves closer to the equilibrium position, the restoring force decreases. At the equilibrium position, the net restoring force vanishes. However, at x = 0, the mass has momentum because of the acceleration that the restoring force has imparted. Therefore, the mass continues past the equilibrium position, compressing the spring. A net restoring force then slows it down until its velocity reaches zero, whereupon it is accelerated back to the equilibrium position again. As long as the system has no energy loss, the mass continues to oscillate. Thus simple harmonic motion is a type of periodic motion. If energy is lost in the system, then the mass exhibits damped oscillation. Note if the real space and phase space plot are not co-linear, the phase space motion becomes elliptical. The area enclosed depends on the amplitude and the maximum momentum. In Newtonian mechanics, for one-dimensional simple harmonic motion, the equation of motion, which is a second-order linear ordinary differential equation with constant coefficients, can be obtained by means of Newton's second law and Hooke's law for a mass on a spring. F n e t = m d 2 x d t 2 = − k x , {\displaystyle F_{\mathrm {net} }=m{\frac {\mathrm {d} ^{2}x}{\mathrm {d} t^{2}}}=-kx,} where m is the inertial mass of the oscillating body, x is its displacement from the equilibrium (or mean) position, and k is a constant (the spring constant for a mass on a spring). Therefore, d 2 x d t 2 = − k m x {\displaystyle {\frac {\mathrm {d} ^{2}x}{\mathrm {d} t^{2}}}=-{\frac {k}{m}}x} Solving the differential equation above produces a solution that is a sinusoidal function: x (t) = c 1 cos ⁡ (ω t) + c 2 sin ⁡ (ω t) , {\displaystyle x(t)=c_{1}\cos \left(\omega t\right)+c_{2}\sin \left(\omega t\right),} where ω = k / m . {\textstyle {\omega }={\sqrt {{k}/{m}}}.} The meaning of the constants c 1 {\displaystyle c_{1}} and c 2 {\displaystyle c_{2}} can be easily found: setting t = 0 {\displaystyle t=0} on the equation above we see that x (0) = c 1 {\displaystyle x(0)=c_{1}} , so that c 1 {\displaystyle c_{1}} is the initial position of the particle, c 1 = x 0 {\displaystyle c_{1}=x_{0}} ; taking the derivative of that equation and evaluating at zero we get that x ˙ (0) = ω c 2 {\displaystyle {\dot {x}}(0)=\omega c_{2}} , so that c 2 {\displaystyle c_{2}} is the initial speed of the particle divided by the angular frequency, c 2 = v 0 ω {\displaystyle c_{2}={\frac {v_{0}}{\omega }}} . Thus we can write: x (t) = x 0 cos ⁡ (k m t) + v 0 k m sin ⁡ (k m t) . {\displaystyle x(t)=x_{0}\cos \left({\sqrt {\frac {k}{m}}}t\right)+{\frac {v_{0}}{\sqrt {\frac {k}{m}}}}\sin \left({\sqrt {\frac {k}{m}}}t\right).} This equation can also be written in the form: x (t) = A cos ⁡ (ω t − φ) , {\displaystyle x(t)=A\cos \left(\omega t-\varphi \right),} where A = c 1 2 + c 2 2 {\displaystyle A={\sqrt {{c_{1}}^{2}+{c_{2}}^{2}}}} tan ⁡ φ = c 2 c 1 , {\displaystyle \tan \varphi ={\frac {c_{2}}{c_{1}}},} sin ⁡ φ = c 2 A , cos ⁡ φ = c 1 A {\displaystyle \sin \varphi ={\frac {c_{2}}{A}},\;\cos \varphi ={\frac {c_{1}}{A}}} or equivalently A = | c 1 + c 2 i | , {\displaystyle A=|c_{1}+c_{2}i|,} φ = arg ⁡ (c 1 + c 2 i) {\displaystyle \varphi =\arg(c_{1}+c_{2}i)} In the solution, c1 and c2 are two constants determined by the initial conditions (specifically, the initial position at time t = 0 is c1, while the initial velocity is c2ω), and the origin is set to be the equilibrium position.[A] Each of these constants carries a physical meaning of the motion: A is the amplitude (maximum displacement from the equilibrium position), ω = 2πf is the angular frequency, and φ is the initial phase.[B] Using the techniques of calculus, the velocity and acceleration as a function of time can be found: v (t) = d x d t = − A ω sin ⁡ (ω t − φ) , {\displaystyle v(t)={\frac {\mathrm {d} x}{\mathrm {d} t}}=-A\omega \sin(\omega t-\varphi),} Speed: ω A 2 − x 2 {\displaystyle {\omega }{\sqrt {A^{2}-x^{2}}}} Maximum speed: v = ωA (at equilibrium point) a (t) = d 2 x d t 2 = − A ω 2 cos ⁡ (ω t − φ) . {\displaystyle a(t)={\frac {\mathrm {d} ^{2}x}{\mathrm {d} t^{2}}}=-A\omega ^{2}\cos(\omega t-\varphi).} Maximum acceleration: Aω2 (at extreme points) By definition, if a mass m is under SHM its acceleration is directly proportional to displacement. a (x) = − ω 2 x . {\displaystyle a(x)=-\omega ^{2}x.} where ω 2 = k m {\displaystyle \omega ^{2}={\frac {k}{m}}} Since ω = 2πf, f = 1 2 π k m , {\displaystyle f={\frac {1}{2\pi }}{\sqrt {\frac {k}{m}}},} and, since T = 1/f where T is the time period, T = 2 π m k . {\displaystyle T=2\pi {\sqrt {\frac {m}{k}}}.} These equations demonstrate that the simple harmonic motion is isochronous (the period and frequency are independent of the amplitude and the initial phase of the motion). Substituting ω2 with k/m, the kinetic energy K of the system at time t is K (t) = 1 2 m v 2 (t) = 1 2 m ω 2 A 2 sin 2 ⁡ (ω t − φ) = 1 2 k A 2 sin 2 ⁡ (ω t − φ) , {\displaystyle K(t)={\tfrac {1}{2}}mv^{2}(t)={\tfrac {1}{2}}m\omega ^{2}A^{2}\sin ^{2}(\omega t-\varphi)={\tfrac {1}{2}}kA^{2}\sin ^{2}(\omega t-\varphi),} and the potential energy is U (t) = 1 2 k x 2 (t) = 1 2 k A 2 cos 2 ⁡ (ω t − φ) . {\displaystyle U(t)={\tfrac {1}{2}}kx^{2}(t)={\tfrac {1}{2}}kA^{2}\cos ^{2}(\omega t-\varphi).} In the absence of friction and other energy loss, the total mechanical energy has a constant value E = K + U = 1 2 k A 2 . {\displaystyle E=K+U={\tfrac {1}{2}}kA^{2}.} An undamped spring–mass system undergoes simple harmonic motion. The following physical systems are some examples of simple harmonic oscillator. A mass m attached to a spring of spring constant k exhibits simple harmonic motion in closed space. The equation for describing the period: T = 2 π m k {\displaystyle T=2\pi {\sqrt {\frac {m}{k}}}} shows the period of oscillation is independent of the amplitude, though in practice the amplitude should be small. The above equation is also valid in the case when an additional constant force is being applied on the mass, i.e. the additional constant force cannot change the period of oscillation. Simple harmonic motion can be considered the one-dimensional projection of uniform circular motion. If an object moves with angular speed ω around a circle of radius r centered at the origin of the xy-plane, then its motion along each coordinate is simple harmonic motion with amplitude r and angular frequency ω. The motion of a body in which it moves to and from a definite point is also called oscillatory motion or vibratory motion. The time period is able to be calculated by T = 2 π l g {\displaystyle T=2\pi {\sqrt {\frac {l}{g}}}} where l is the distance from rotation to the object's center of mass undergoing SHM and g is gravitational acceleration. This is analogous to the mass-spring system. A pendulum making 25 complete oscillations in 60 s, a frequency of 0.416 Hertz In the small-angle approximation, the motion of a simple pendulum is approximated by simple harmonic motion. The period of a mass attached to a pendulum of length l with gravitational acceleration g {\displaystyle g} is given by T = 2 π l g {\displaystyle T=2\pi {\sqrt {\frac {l}{g}}}} This shows that the period of oscillation is independent of the amplitude and mass of the pendulum but not of the acceleration due to gravity, g {\displaystyle g} , therefore a pendulum of the same length on the Moon would swing more slowly due to the Moon's lower gravitational field strength. Because the value of g {\displaystyle g} varies slightly over the surface of the earth, the time period will vary slightly from place to place and will also vary with height above sea level. This approximation is accurate only for small angles because of the expression for angular acceleration α being proportional to the sine of the displacement angle: − m g l sin ⁡ θ = I α , {\displaystyle -mgl\sin \theta =I\alpha ,} where I is the moment of inertia. When θ is small, sin θ ≈ θ and therefore the expression becomes − m g l θ = I α {\displaystyle -mgl\theta =I\alpha } which makes angular acceleration directly proportional and opposite to θ, satisfying the definition of simple harmonic motion (that net force is directly proportional to the displacement from the mean position and is directed towards the mean position). Main article: Scotch yoke A Scotch yoke mechanism can be used to convert between rotational motion and linear reciprocating motion. The linear motion can take various forms depending on the shape of the slot, but the basic yoke with a constant rotation speed produces a linear motion that is simple harmonic in form. Scotch yoke animation Circle group Complex harmonic motion Damping ratio Harmonic oscillator Isochronous timing Lorentz oscillator model Newtonian mechanics Pendulum Rayleigh–Lorentz pendulum Small-angle approximation String vibration Uniform circular motion ^ The choice of using a cosine in this equation is a convention. Other valid formulations are: x (t) = A sin ⁡ (ω t + φ ′) , {\displaystyle x(t)=A\sin \left(\omega t+\varphi '\right),} where tan ⁡ φ ′ = c 1 c 2 , {\displaystyle \tan \varphi '={\frac {c_{1}}{c_{2}}},} since cos θ = sin(π/2 − θ). ^ The maximum displacement (that is, the amplitude), xmax, occurs when cos(ωt ± φ) = 1, and thus when xmax = A. ^ "Simple harmonic motion | Formula, Examples, & Facts | Britannica". britannica.com. 2024-09-30. Retrieved 2024-10-11. ^ "Simple Harmonic Motion – Concepts". Fowles, Grant R.; Cassiday, George L. (2005). Analytical Mechanics (7th ed.). Thomson Brooks/Cole. ISBN 0-534-49492-7. Taylor, John R. (2005). Classical Mechanics. University Science Books. ISBN 1-891389-22-X. Thornton, Stephen T.; Marion, Jerry B. (2003). Classical Dynamics of Particles and Systems (5th ed.). Brooks Cole. ISBN 0-534-40896-6. Walker, Jearl (2011). Principles of Physics (9th ed.). Hoboken, New Jersey: Wiley. ISBN 978-0-470-56158-4. Wikimedia Commons has media related to Simple harmonic motion. Simple Harmonic Motion from HyperPhysics Java simulation of spring-mass oscillator Geogebra applet for spring-mass, with 3 attached PDFs on SHM, driven/damped oscillators, spring-mass with friction Retrieved from " Download the Testbook APP & Get Pass Pro Max FREE for 7 Days10,000+ Study NotesRealtime Doubt Support71000+ Mock TestsRankers Test Series+ more benefitsDownload App Now By the end of this section, you will be able to: Describe a simple harmonic oscillator. Explain the link between simple harmonic motion and waves. The oscillations of a system in which the net force can be described by Hooke's law are of special importance, because they are very common. They are also the simplest oscillatory systems. Simple Harmonic Motion (SHM) is the name given to oscillatory motion for a system where the net force can be described by Hooke's law, and such a system is called a simple harmonic oscillator. If the net force can be described by Hooke's law and there is no damping (by friction or other non-conservative forces), then a simple harmonic oscillator will oscillate with equal displacement on either side of the equilibrium position, as shown for an object on a spring in Figure 1. The maximum displacement from equilibrium is called the amplitude X. The units for amplitude and displacement are the same, but depend on the type of oscillation. For the object on the spring, the units of amplitude and displacement are meters; whereas for sound oscillations, they have units of pressure (and other types of oscillations have yet other units). Because amplitude is the maximum displacement, it is related to the energy in the oscillation. Figure 1. An object attached to a spring sliding on a frictionless surface is an uncomplicated simple harmonic oscillator. When displaced from equilibrium, the object performs simple harmonic motion that has an amplitude X and a period T. The object's maximum speed occurs as it passes through equilibrium. The stiffer the spring is, the smaller the period T. The greater the mass of the object is, the greater the period T. Find a bowl or basin that is shaped like a hemisphere on the inside. Place a marble inside the bowl and tilt the bowl periodically so the marble rolls from the bottom of the bowl to equally high points on the sides of the bowl. Get a feel for the force required to maintain this periodic motion. What is the restoring force and what role does the force you apply play in the simple harmonic motion (SHM) of the marble? What is so significant about simple harmonic motion? One special thing is that the period T and frequency f of a simple harmonic oscillator are independent of amplitude. The string of a guitar, for example, will oscillate with the same frequency whether plucked gently or hard. Because the period is constant, a simple harmonic oscillator can be used as a clock. Two important factors do affect the period of a simple harmonic oscillator. The period is related to how stiff the system is. A very stiff object has a large force constant k, which causes the system to have a smaller period. For example, you can adjust a diving board's stiffness—the stiffer it is, the faster it vibrates, and the shorter its period. Period also depends on the mass of the oscillating system. The more massive the system is, the longer the period. For example, a heavy person on a diving board bounces up and down more slowly than a light one. In fact, the mass m and the force constant k are the only factors that affect the period and frequency of simple harmonic motion. The period of a simple harmonic oscillator is given by [latex]T=2\pi\sqrt{\frac{m}{k}}\\[/latex] and, because [latex]f=\frac{1}{T}\\[/latex], the frequency of a simple harmonic oscillator is [latex]f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}\\[/latex]. Note that neither T nor f has any dependence on amplitude. Find two identical wooden or plastic rulers. Tape one end of each ruler firmly to the edge of a table so that the length of each ruler that protrudes from the table is the same. On the free end of one ruler tape a heavy object such as a few large coins. Pluck the ends of the rulers at the same time and observe which one undergoes more cycles in a time period, and measure the period of oscillation of each of the rulers. If the shock absorbers in a car go bad, then the car will oscillate at the least provocation, such as when going over bumps in the road and after stopping (See Figure 2). Calculate the frequency and period of these oscillations for such a car if the car's mass (including its load) is 900 kg and the force constant (k) of the suspension system is 6.53 × 104 N/m. Figure 2. The bouncing car makes a wavelike motion. If the restoring force in the suspension system can be described only by Hooke's law, then the wave is a sine function. (The wave is the trace produced by the headlight as the car moves to the right.) Strategy The frequency of the car's oscillations will be that of a simple harmonic oscillator as given in the equation [latex]f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}\\[/latex]. The mass and the force constant are both given. Solution Enter the known values of k and m: [latex]\displaystyle{f}=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{6.53\times10^4\text{ N/m}}{900\text{ kg}}}\\[/latex] Calculate the frequency: [latex]\frac{1}{2\pi}\sqrt{72.6/\text{s}^{-2}}=1.3656/\text{s}^{-1}\approx1.36/\text{s}^{-1}=1.36\text{ Hz}\\[/latex] You could use [latex]T=2\pi\sqrt{\frac{m}{k}}\\[/latex] to calculate the period, but it is simpler to use the relationship [latex]T=\frac{1}{f}\\[/latex] and substitute the value just found for f: [latex]\displaystyle{T}=\frac{1}{f}=\frac{1}{1.356\text{ Hz}}=0.738\text{ s}\\[/latex] Discussion The values of T and f both seem about right for a bouncing car. You can observe these oscillations if you push down hard on the end of a car and let go. The Link between Simple Harmonic Motion and Waves Figure 3. The vertical position of an object bouncing on a spring is recorded on a strip of moving paper, leaving a sine wave. If a time-exposure photograph of the bouncing car were taken as it drove by, the headlight would make a wavelike streak, as shown in Figure 2. Similarly, Figure 3 shows an object bouncing on a spring as it leaves a wavelike "trace of its position on a moving strip of paper. Both waves are sine functions. All simple harmonic motion is intimately related to sine and cosine waves. The displacement as a function of time t in any simple harmonic motion—that is, one in which the net restoring force can be described by Hooke's law, is given by [latex]x(t)=X\cos\frac{2\pi{t}}{T}\\[/latex], where X is amplitude. At t = 0, the initial position is x0 = X, and the displacement oscillates back and forth with a period T. (When t = T, we get x = X again because cos 2π = 1.). Furthermore, from this expression for x, the velocity v as a function of time is given by [latex]v(t)=-v_{\text{max}}\sin\left(\frac{2\pi{t}}{T}\right)\\[/latex], where [latex]v_{\text{max}}=\frac{2\pi{X}}{T}=X\sqrt{\frac{k}{m}}\\[/latex]. The object has zero velocity at maximum displacement—for example, v=0 when t=0, and at that time x=X. The minus sign in the first equation for v(t) gives the correct direction for the velocity. Just after the start of the motion, for instance, the velocity is negative because the system is moving back toward the equilibrium point. Finally, we can get an expression for acceleration using Newton's second law. [Then we have x(t), v(t), t, and a(t), the quantities needed for kinematics and a description of simple harmonic motion.] According to Newton's second law, the acceleration is [latex]a=\frac{F}{m}=\frac{kx}{m}\\[/latex]. So, a(t) is also a cosine function: [latex]a(t)=-\frac{kX}{m}\cos\frac{2\pi{t}}{T}\\[/latex]. Hence, a(t) is directly proportional to and in the opposite direction to a(t). Figure 4 shows the simple harmonic motion of an object on a spring and presents graphs of x(t), v(t), and a(t) versus time. Figure 4. Graphs of and versus t for the motion of an object on a spring. The net force on the object can be described by Hooke's law, and so the object undergoes simple harmonic motion. Note that the initial position has the vertical displacement at its maximum value X; v is initially zero and then negative as the object moves down; and the initial acceleration is negative, back toward the equilibrium position and becomes zero at that point. The most important point here is that these equations are mathematically straightforward and are valid for all simple harmonic motion. They are very useful in visualizing waves associated with simple harmonic motion, including visualizing how waves add with one another. Suppose you pluck a banjo string. You hear a single note that starts out loud and slowly quiets over time. Describe what happens to the sound waves in terms of period, frequency and amplitude as the sound decreases in volume. Solution Frequency and period remain essentially unchanged. Only amplitude decreases as volume decreases. Part 2 A babysitter is pushing a child on a swing. At the point where the swing reaches x, where would the corresponding point on a wave of this motion be located? Solution x is the maximum deformation, which corresponds to the amplitude of the wave. The point on the wave would either be at the very top or the very bottom of the curve. A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energy for each spring. Click to run the simulation. Selected Solutions Simple harmonic motion is oscillatory motion for a system that can be described only by Hooke's law. Such a system is also called a simple harmonic oscillator. Maximum displacement is the amplitude X. The period T and frequency f of a simple harmonic oscillator are given by [latex]T=2\pi\sqrt{\frac{m}{k}}\\[/latex] and [latex]f=\frac{1}{2\pi }\sqrt{\frac{k}{m}}\\[/latex] , where m is the mass of the system. Displacement in simple harmonic motion as a function of time is given by [latex]x\left(t\right)=X\text{cos}\frac{2\pi{t}}{T}\\[/latex]. The velocity is given by [latex]v\left(t\right)=-{v}_{\text{max}}\text{sin}\frac{2\pi{t}}{T}\\[/latex], where [latex]{v}_{\text{max}}=\sqrt{\frac{k}{m}}X\\[/latex]. The acceleration is found to be [latex]a(t)=-\frac{kX}{m}\cos{2\pi{t}}{T}\\[/latex]. Conceptual Questions What conditions must be met to produce simple harmonic motion? (a) If frequency is not constant for some oscillation, can the oscillation be simple harmonic motion? (b) Can you think of any examples of harmonic motion where the frequency may depend on the amplitude? Give an example of a simple harmonic oscillator, specifically noting how its frequency is independent of amplitude. Explain why you expect an object made of a stiff material to vibrate at a higher frequency than a similar object made of a spongy material. As you pass a freight truck with a trailer on a highway, you notice that its trailer is bouncing up and down slowly. Is it more likely that the trailer is heavily loaded or nearly empty? Explain your answer. Some people modify cars to be much closer to the ground than when manufactured. Should they install stiffer springs? Explain your answer. Problems & Exercises A type of cuckoo clock keeps time by having a mass bouncing on a spring, usually something cute like a cherub in a chair. What force constant is needed to produce a period of 0.500 s for a 0.0150-kg mass? If the spring constant of a simple harmonic oscillator is doubled, by what factor will the mass of the system need to change in order for the frequency of the motion to remain the same? A 0.500-kg mass suspended from a spring oscillates with a period of 1.50 s. How much mass must be added to the object to change the period to 2.00 s? By how much leeway (both percentage and mass) would you have in the selection of the mass of the object in the previous problem if you did not wish the new period to be greater than 2.01 s or less than 1.99 s? Suppose you attach the object with mass m to a vertical spring originally at rest, and let it bounce up and down. You release the object from rest at the spring's original rest length. (a) Show that the spring exerts an upward force of 2.00 mg on the object at its lowest point. (b) If the spring has a force constant of 10.0 N/m and a 0.25-kg-mass object is set in motion as described, find the amplitude of the oscillations. (c) Find the maximum velocity. A diver on a diving board is undergoing simple harmonic motion. Her mass is 55.0 kg and the period of her motion is 0.800 s. The next diver is a male whose period of simple harmonic oscillation is 1.05 s. What is his mass if the mass of the board is negligible? Suppose a diving board with no one on it bounces up and down in a simple harmonic motion with a frequency of 4.00 Hz. The board has an effective mass of 10.0 kg. What is the frequency of the simple harmonic motion of a 75.0-kg diver on the board? The device pictured in Figure 6 entertains infants while keeping them from wandering. The child bounces in a harness suspended from a door frame by a spring constant. Figure 6. This child's toy relies on springs to keep infants entertained. (credit: By Humboldthead, Flickr) (a) If the spring stretches 0.250 m while supporting an 8.0-kg child, what is its spring constant? (b) What is the time for one complete bounce of this child? (c) What is the child's maximum velocity if the amplitude of her bounce is 0.200 m? A 90.0-kg skydiver hanging from a parachute bounces up and down with a period of 1.50 s. What is the new period of oscillation when a second skydiver, whose mass is 60.0 kg, hangs from the legs of the first, as seen in Figure 7. Figure 7. The oscillations of one skydiver are about to be affected by a second skydiver. (credit: U.S. Army, www.army.mil) amplitude: the maximum displacement from the equilibrium position of an object oscillating around the equilibrium position simple harmonic motion: the oscillatory motion in a system where the net force can be described by Hooke's law simple harmonic oscillator: a device that implements Hooke's law, such as a mass that is attached to a spring, with the other end of the spring being connected to a rigid support such as a wall 1. 2.37 N/m 3. 0.389 kg 6. 94.7 kg 9. 1.94 s